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Investigating Students' Understanding of Complex Numbers and Its Relation to Algebraic Groups Using Psychometric Test and APOS Theory

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ABSTRACT

This study focused on students' inability to correctly use psychometric tests applied for complex numbers in the context of generating an Algebraic abelian group (closure, commutative, associatively, inverse, and identity). This was done using a psychometric test (Kuder and Richardson formula) and APOS Theory after they had been taught the relevant content. The 41 participants were second-year students in the Department of Statistics at the Federal Polytechnic Ile-Oluji. They were given a class test comprising eleven (11) objective questions, and 4 questions relating to group theory were extracted for this analysis. After that, six (6) students were interviewed to gain insight into their perspectives. It was observed that most of the students had difficulty understanding the concept of algebraic groups about complex numbers, founded on not seeing the relevance in the prerequisite knowledge of an algebraic group. Also, if the reliability test of the question is unacceptable, then a good percentage of the participant lacked the mental construction relating to APOS to answer such a question.

Keywords: *Complex Number, Algebraic group, Prerequisite, Psychometrics, APOS Theory.*

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INTRODUCTION

Complex numbers are numbers that can be expressed in the form $p + iq$, where p and q are real numbers, and i is related to the solution of the equation $x^2 = -1$, ($i = \sqrt{-1}$). Because no real number satisfies this equation, i is called an imaginary number. For the complex number $p + iq$, p is called the real part, and q is called the imaginary part. It is generated from the solution of quadratic equation $qx^2 + px + r = 0$ where $x = \frac{-p \pm \sqrt{p^2 - 4qr}}{2q}$ such that $p^2 < 4qr$. The application of complex numbers is important in fields related to mathematics, since it is related to other concepts. For example: quadratic equation, matrices and derivative complex. Therefore, one could conclude that they are the building blocks of more intricate mathematics, such as algebra. Complex Numbers can be applied to many aspects of real life, especially in the areas of electronics and electromagnetism. Complex numbers have some useful mathematical properties that actually make life easier when modeling systems with sinusoidal inputs and geometrical applications. This emphasizes the importance of complex numbers (See Steve Howell(2010)).

In an attempt to unify complex numbers and algebraic group, the research of Donu Arapura, (2012), combined the study of algebraic geometry with differential and complex geometry and unified these subjects using sheaf-theoretic ideas. It was also an ideal text for showing students the connections between algebraic geometry, complex geometry, and topology, and it brought the reader close to the forefront of research in Hodge theory and related fields. Ricardo Karam, (2020), posited that complex numbers are broadly used in physics, as a calculation tool, as it makes things easier due to Euler's formula. In the end, it is only the real component that has physical meaning or the two parts (real and imaginary) are treated separately as real quantities. As much as it is perceived to make things easier, students find it difficult and in terms of approach to study, Areti Panaoura, Iliada Elia, et al, 2006, revealed via a comparative study that a common phenomenon was students' difficulty in complex number problem solving, irrespective of their preferred type of approach. The study revealed that geometric approach is used frequently while algebraic approach is used consistently and in a more persistent way.

Whereas, in the study of Morton Hamermesh, 2005, the author revealed that Group Theory is a mathematical technique for dealing with issues of symmetry. Having had to deal with symmetry for decades, the study of group theory is a recent one. Group theory is the basic tool used in solving difficult problems. The strength of group theory lies in the ability to translate many complex numbers into very simple linear algebra, Mildred S Dresselhaus, et al, 2007. According to Nathan Carter, 2021, Group theory is a part of mathematics that studies symmetry, arts, music, architecture and many other contexts. The author believed that the beauty of group theory, which seeks to simplify, is lost when it is taught in a technical way and difficult to understand for the students. Due to the importance and student perspective towards complex number and group theory, it is therefore important to analyze the understanding of students towards the basic concept of complex numbers, especially, as it connects to algebraic group, so as to come up with possible solutions to the difficulties encountered by students.

BACKGROUND OF THE STUDY

For the benefit of the reader we note the following basic definitions and theorem.

Definition1: Semi-group could be defined as an algebraic structure of a set equipped with associative internal binary operation satisfying certain properties similar to but less stringent than those of a group theory, that is, an algebraic structure which is closed and whose operation is associative. An example is a set of positive integers with multiplication $(' * ')$ as the operation.

If $(S,*)$ is a semi-group, then an element $\varepsilon_1 \in S$ is called a left unit or left identity of S and $\varepsilon_r \in S$ is the right unit or right identity of S, for all $a \in S$, if $\varepsilon_1 * a = a = a * \varepsilon_r$. If $\varepsilon_r = \varepsilon_1 = \varepsilon$ which is the left identity, the right identity is, $\varepsilon \in S$, $\varepsilon * a = a * \varepsilon = a$, for all $a \in S$.. Hence a semi-group with identity is called monoid. Examples are: integer, natural and complex number; that is,$(\mathbb{Z},*)(\mathbb{N},*)(\mathbb{C},*)$ respectively.

Definition 2: Let $(S,*)$be a monoid and $a \in S$. An element $b \in S$ is called a left inverse of a if $b * a = \varepsilon$. Call $b \in S$ a right inverse of a if $a * b = \varepsilon$. An element b which is both right and left inverse of a is called the inverse of a. A monoid in which every element has an inverse is called a **group**. i.e.$(\mathbb{C} - \{0\},*)$ (see Kuku (2010))

Theorem 1: Let a be an element of a group $(G,*)$. If b is a right inverse of a and c a left inverse of $a, then\ b = c$. So the inverse of any element of a group G is unique.

Definition 3: A group $(G,*)$is said to be commutative or abelian if $a * b = b * a$ for all $a, b \in G$.

BACKGROUND ON COMPLEX NUMBER AS GROUP

Complex numbers as abelian algebraic group:

Given two complex numbers C_1 , C_2 such that $C_1 = (p_1 + iq_1)$, $C_2 = (p_2 + iq_2)$and

$C_3 = (p_3 + iq_3)$then note the following in the context of multiplication or product(s) of complex numbers:

- $C_1 * C_2 = (p_1 + iq_1) \times (p_2 + iq_2) = p_1p_2 + ip_1q_2 + iq_1p_2 - q_1q_2$ since
  $$i \times i = \sqrt{-1} \times \sqrt{-1} = -1$$
  $= p_1p_2 - q_1q_2 + ip_1q_2 + ip_2q_1 = (p_1p_2 - q_1q_2) + i(p_1q_2 + p_2q_1) = C_2 * C_1$ ….. [1]

It is obvious to say that $(p_1p_2 - q_1q_2) = F$ is the real part, while $(p_1q_2 + p_2q_1) = G$ is the imaginary part, therefore: $C_1C_2 = F + iG$

The associative property is obvious, since $C_1 * (C_2 * C_3) = (C_1 * C_2) * C_3$ …….. [2]

- $(C_1 * C_2) * C_3 = \big((p_1 + iq_1) \times (p_2 + iq_2)\big) \times (p_3 + iq_3)$

$= (p_1p_2 - q_1q_2 + ip_1q_2 + ip_2q_1)(p_3 + iq_3)$

$= (p_1p_2p_3 - q_1q_2p_3 + ip_1p_3q_2 + ip_2p_3q_1 + ip_1p_2q_3 - iq_1q_2q_3 - p_1q_2q_3 - p_2q_1q_3)$

$= (p_1 + iq_1) \times (p_2p_3 - q_2q_3 + ip_2q_3 + ip_3q_2) = C_1 * (C_2 * C_3)$

Inverse and Identity: If $C = p + iq$, then its inverse is given as C^{-1}

$$C^{-1} = \frac{1}{C} = \frac{1}{p+iq} = \frac{p-iq}{p^2+q^2} = \frac{p}{p^2+q^2} - i\frac{q}{p^2+q^2} = H + iJ \quad \ldots\ldots\ldots [3]$$

It is trivial that $C * C^{-1} = 1 = 1 + i0 \quad \ldots\ldots..[4]$, serves as the identity of complex number and it is closed.

All principles of an abelian group (closure, commutativity, associativity, identity and inverse) could be determined by understanding the principles of common operator (Addition, multiplication (product) and quotient (division)) on complex numbers. Since all results shown above are complex numbers then it was closed under addition and multiplication (closure).

In the curriculum under review, which is a first year National Diploma curriculum, the design entails the teaching of Algebraic Group in the first semester, seemingly to prepare the students for proper usage of the algebraic group application in other sectors, including the establishment of the relationship with complex numbers.

In order to factor out reasons behind the idea of prerequisite and the students' understanding of the concept, it is vital to make recourse to literatures with a view of understanding the opinions of researchers in the said area, as it affects mathematical prerequisite courses. Terry, N. B et al(2016) , said it is vital to measure the scores of students below average in prerequisite courses, for the purpose of measuring understanding. Study revealed that prerequisite knowledge, rather than intelligence plays a great role in subsequent learning, on the other hand, Valstar, S. et al (2019), showed that focus should be placed on the proficiency of the prerequisite courses, rather than the success, that is, the strength and impact of the previous course on the later one. Result showed that prerequisite proficiency on entry to the course was low, but moderately increased during the term. It is therefore important to ensure that the students adequately know that it is vital to understand the importance of prerequisite courses for proper planning in their study routine ahead of the later course.

Considering the fact that mathematics can as well be a prerequisite to other course fields, it is imperative to guarantee that the students prior to the entrance into the course structure are well taken through the route of prerequisite knowledge, impact, advantages and disadvantages, as well as the strength. McCarron & Burstein, (2017)., posits that mathematics has long served as a prerequisite to introductory financial accounting in the college

business curriculum. The result of the research revealed that the Grade Point Average (GPA) of students at the end of the programme improves when the prerequisite of mathematics and English are considered.
According to Baranyi & Molonta. (2021)., curriculum prerequisite plays a vital role in shaping an Institution's programs. Many researchers have threaded the part of understanding the suitable route to design the knowledge of prerequisite learning goals, this is most probably because of the importance of this concept to students, programme design and tutors, as it suggests leaning goals, completion time and drop out level. Islam, F. et al. (2008), examined the value of prerequisite course for statistics. In the research, the researchers observed that business schools make it compulsory to have prerequisite courses in order to aid the preparedness of students for core statistical courses. An area of interest in the research is the fact that students are allowed to freely choose their prerequisite courses, which in turn corroborate the idea of having prerequisite courses as preparatory courses for major courses. However, the researchers discovered that despite this subtle platform of choosing the preparatory courses, students still have difficulty in graduating from the courses, but, served as a learning platform for the students, who can be helped in terms of grades by the Institution through marginal scoring. Hull et al(2016), is of the opinion that the successful completion of a programme is determined by various factors, among which is the prerequisite knowledge of the students.

1.2 APOS AND $P_{KR_{20}}$

This research sought to analyze the result of the collected data, through the Test and Oral Interview with the APOS Theory. APOS (action-process-object-schema) Theory, a constructivist theory, posits that an individual's understanding of a mathematical topic develops through reflecting on problems and their solutions in a social context, constructing or reconstructing certain mental structures and organizing these into schemas to deal with problematic situations. The main ideas of APOS Theory were introduced by Dubinsky, . (1984).. The acronym APOS was first used by Cottrill, et al (1996). It has been a relevant tool by researchers, used to analyze students' understanding of various mathematical topics Maharaj & Ntuli (2018).An individual's mathematical knowledge increases his/her tendency to respond to perceived mathematical problem situations and their solutions Maharaj (2018). This implies that an individual's acquisition of mathematical knowledge is the construction and even reconstruction of mental structures for use, when faced with a problem situation.

Meanwhile, The Kuder-Richardson Formula 20($P_{KR_{20}}$) is a measure for internal consistency reliability. Reliability refers to how consistent the results from the test are, or how well the test is actually measuring what you want it to measure. Essentially it lets you know whether the test as a whole discriminates among students who mastered the subject matter and those who did not. The closer the $P_{KR_{20}}$ is to +1.0 the more reliable a test is considered, because the questions consistently discriminates among higher and lower performing students while $P_{KR_{20}}$ of 0.0 or close to 0.0 means the test questions did not discriminate at all (see Hull, et al (2016), *Ritter (2010))*. The $P_{KR_{20}}$ is used for items that have varying difficulty. For example, some items might be very easy, others more challenging. It should only be used if there is a correct answer for each question.

CONCEPTUAL FRAMEWORK

This study was guided by some principles and literature reviews as discussed by Maharaj (2018).

1 There is a perquisite knowledge in the body of mathematics, which informed the formulation of students' expected learning outcomes and development of simple diagnostic questions;

2 Students' response in solving complex numbers in relations to algebraic group concept gives an insight into their understanding of the rule of complex numbers and principles of algebraic group;

3 The quantitative data collected from the relevant quizzes, which focus on students' responses to problems on complex numbers in relations to algebraic group concept would reveal trends that could be used to inform teaching with the aim of improving students' understanding of complex numbers and to keep informed on perquisite knowledge.

GENETIC DECOMPOSITION

As discussed by Cottrill et al (1996), an action is a transformation of objects perceived by an entity as principally external and as taking, either explicitly or from memory, step-by-step instructions on how to perform the operation. When an action is repeated and the entity reflects upon it, he or she can make an internal mental construction called a process which the individual can think of as performing the same kind of action, but no longer with the need of external stimuli. An individual can think of performing a process without actually doing it, and therefore can think about reversing it and composing it with other processes. An object is constructed from a process when the individual becomes aware of the process as a totality and realizes that transformations can act on it. Finally, a schema for a certain mathematical concept is an individual's collection of actions, processes, objects, and other schemas which are linked by some general principles to form a framework in the individual's mind that may be brought to bear upon a problem situation involving that concept

The ultimate goal of a researcher is to break down theories and concepts to the basic, for maximum understanding. Genetic decomposition is a line of breaking down concepts and transforming those concepts, with the aim of coming up with or forming new concepts. It is a term used for solving various teaching problems, so as to give room for instructional decisions. The stages of an APOS for the complex number concept are presented here in a hierarchical, ordered list. This will help in explaining the constructions and, in some sense, each conception in the list must be constructed before the next step is possible.

APOS theory could be explained on complex number on student level of understanding as:

Action: having the base knowledge on complex number $\sqrt{p^2 - 4qr}$ *with* $p^2 < 4qr$, where $\sqrt{-1} = i$ and to recognize one if seen.

Process: level of understanding that involve manipulations of operator (addiction, subtraction, multiplication and division) of complex numbers. Recognizing the context in which the operations occur and knowing what to do; these could be mentally done or there could be physical evidence.

Object: level of understanding that involve properties that made up an algebraic abelian group (closure, associative, identity, inverse, commutative). Manipulations of operators to confirm properties of the concept.

Schema: understanding the principle of complex number concept and be able to relate it with another mathematical concept i.e algebraic group. This would include a framework of the necessary actions, processes and objects with the connections between them to use when confronted with a problem-solving situation based on complex numbers and algebraic group theory.

It should be understood that the knowledge constructed in a stage will impact on the next stage. Although, when a student or an individual is developing her or his understanding of a concept, the constructions of APOS are not actually made in such a linear manner. i.e a student that has Schema on collection of team principle in mathematic might work on the process level, for example in$\left((2-3i)+2(5+6i)=12+9i\right)$, without recognizing the function as complex$\left(\sqrt{-1}=i\right)$. For better explanation and to know which level of construction needed to be worked on or spend more time on to explain to the students, we shall follow the structure in a linear mode in respect to the fact that in attending Object and Schema level of understanding of a concept-relevant perquisites are needed to be understood.

METHODOLOGY AND PARTICIPANTS

Mathematical Method I is a second-year course in the Department of Statistics at the Federal Polytechnic Ile-Oluji, Ondo State, Nigeria, wherein Complex Numbers is a topic. The researchers set 11 objective questions with four options (a to d, with only one correct option per question) based on the concept of complex numbers, which was taught in the second year , 4 questions(5,7,8 and 11) relating to group properties were extracted(as question 1,2,3 and 4 respectively) for this research. The concept of group was taught in the first-year course titled: 'Logic and linear algebra, which was passed by all students tested. Each of the questions had a weighting of 1 mark; so the possible score for a student could range from 0 to 4 for the basis of this research. A total number of 41 second-year diploma students was registered for the course at the Federal Polytechnic Ile-Oluji, where Complex Numbers as a topic is compulsory for students pursuing a National Diploma in Statistics. The test was administered to the students as a classroom test with invigilators present in each classroom. The test involved the use of two (2) classrooms. The duration given for the test was 40 minutes.

That test was administered after students were exposed to formal lectures on the required concept in Complex Numbers. By taking that test, students could determine their strength and weakness on the topic before sitting for the examination and students were expected to take remedial action; that is revise or seek for help where needed. Furthermore, the insights from the test results were used by the tutor as a guide when revising the course with the students and to determine the students' level of understanding complex numbers in relation to algebraic group.

The use of difficulty Index and discrimination index were also analyzed. Difficulty Index is the proportion or probability that candidates or students will answer a test item correctly.

$$Difficulty\ index = \frac{Total\ correct\ answer\ per\ question}{Total\ number\ of\ students} \times 100$$

Generally, more difficult items have a lower percentage, or P-value (see Table 1 below). Discrimination index is a measure of how well an item distinguishes between examinees who are knowledgeable and those who are not (see Table 2) [10]. There are actually several ways to compute an item discrimination, but one of the most common ways is the point-biserial correlation.

Table 1. Interpretation of the Difficulty Index Based on Students' Mean Percentage Score

Difficulty index	Interpretation
5 ≤	Extremely difficult or something wrong with the question
6-10	Very difficult
11-20	Difficult
20-34	Moderately difficult
35-60	About right for the average student
60-80	Fairly easy
81-89	Easy
90-94	Very easy
95-100	Extremely easy

Source: Maharaj A and Ntuli M (2018)

Table 2. Interpretation of the Discrimination Index (*100)

Discrimination index	Interpretation
Negative	Question probably invalid
20-29	Weak discrimination
30-49	Adequate discrimination
50 and above	Very good discrimination

Source: Maharaj A and Ntuli M (2018)

Table 3. Interpretation of the Kuder and Richardson formula 20 ($P_{KR\ 20}$) reliability

Stages	$P_{KR\ 20}$ VALUES	Interpretation
1	≥0.9	Excellent
2	0.8 – 0.89	Good
3	0.7 - 0.79	Average
4	0.6 - 0.69	Questionable
5	0.5 – 0.59	Poor
6	<0.5	Unacceptable

In furtherance of the data analysis, six (6) of the tested students, who scored marks below average, were subjected to oral examination, with the basic motive of deducing the justification behind the answers given to the questions tested upon, via probing questions and the students' responses were noted by the researchers.

The student participants signed a consent form and ethical clearance for the study was obtained from the Rector through the Head of the Department.

RESULTS AND DISCUSSION

The central concept(s) focused on by the 11 questions were classified as follows: Quadratic expression leading to complex number $p^2 < 4qr$; Argand diagram representation; Modulus of complex number; Argument and trigonometric function; Commutative concept of complex numbers; Distribution concept of complex number; Inverse concept of complex numbers; Identity concept of complex numbers; product of polar form of complex numbers; Quotient of polar form of complex numbers; Associative concept of complex numbers. In this section, for easy readability and to examine students understanding of complex number in relation to algebraic group, 4 questions relating to group properties 5,7,8 and 11 were extracted as question 1,2,3 and 4 respectively and analyzed (include dare relevant extracts from the interviews, with question structure analysis aimed at difficulty index, discriminating index and reliability of the questions).

Table 4. Mark distribution (n = 41)

Mark	0	1	2	3	4
Mark Frequency	8	10	15	7	1

source: field survey

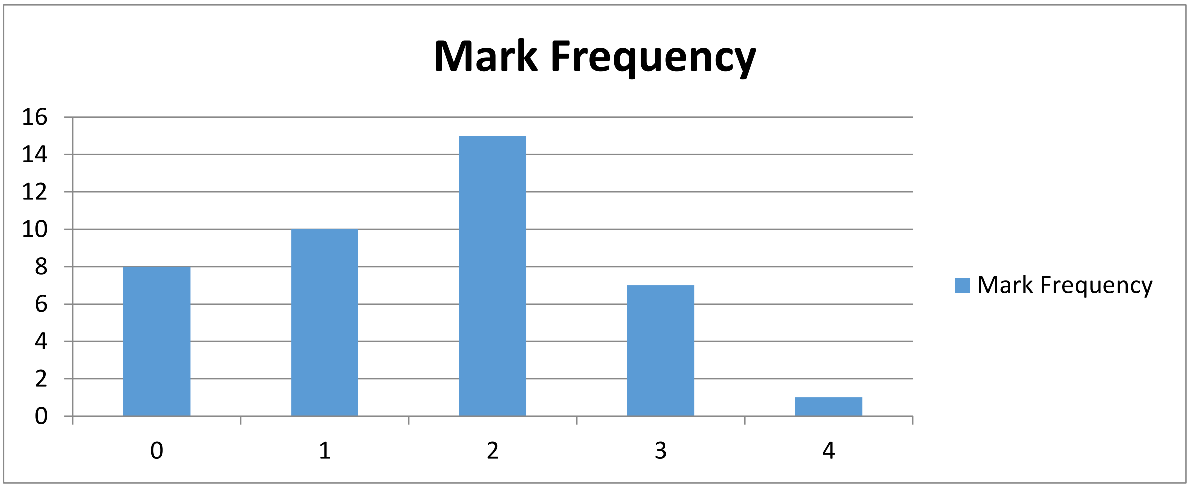

Figure 1 is a Bar Chart showing the students' mark frequency distribution from 0-4 generated from Table 3.

The Kuder and Richardson formula 20 is given by

$$P_{KR_{20}} = \frac{K}{K-1}\left(1 - \frac{\sum_{j=1}^{K} P_j Q_j}{\sigma^2}\right)$$

where

K= number of questions

Sum of all PQ's (P = probability of correct answer, Q = probability of wrong answer)

σ^2= variance of the total scores of all the people taking the test.

The average value of difficulty index shows the test was about right for any average student and there was an very good discrimination among the student with discrimination value of 50 (interpretation from Tables 1, 2 and 3). The value of the $P_{KR_{20}}$ which is 0.38 showed the reliability of the test is unacceptable (interpretation from Tables 2.1) and s inconsistent , which was suitable for the research objectives; students' ability to correctly apply complex numbers solving principle in generating an Algebraic group (closure, commutative, associativity, inverse and identity), also to assess and measure their understanding of complex numbers concept based on their curriculum. This brought the high possibility of generalizing that the challenges faced by a student are common to most of the students.

Question 1. Commutative Concept Complex Number

5. If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, what statement verify commutative rule of complex number ?

a) $(z_1 - z_2) = (z_2 + z_1)$
b) $(z_1 * z_2) = (z_2 * z_1)$
c) $(z_1 * z_2) = (z_2 \div z_1)$
d) $(z_1 - z_2) = (z_2 - z_1)$

Table 6. Option distribution and indices for Question 1 (n = 41)

Question	a	**b***	c	d	No Response	Difficulty index	Discriminating index (*100)
1	6	26	3	6	0	63.41	67

Source: field survey

Table 6 shows the option distribution frequency on question 1 as chosen by the students.

The difficulty index of question 1 is fairly easy (see Table 1) while the discrimination index suggested that there is a very good discrimination among able students from those who were less able to correctly answer this question (see Table 2).This concept is known by the students since majority picked the correct option, although some still found it confusing during the interview when asked 'what do you understand by commutative concept of complex numbers and why did you pick your option?

S2- something like this was taught in our tutorial but he was unable to finish it so I just picked it

S3- Yes, we have been taught before, but am not sure, the left must be equal to right

S6: cumulative? I do hear it.

We observed that S6 did not have an understanding of the commutative concept and also confusing the concept (by pronouncing commutative as cumulative). Students S2 and S3 were conversant with the concept, but were not equipped with the process understanding of the concept as was observed. Commutative is a known concept by the class which is one of the properties of an algebraic group about 36% of the participants lacked the schema in relating complex number $(\mathbb{C} - \{0\},*)$ with abelian algebraic group . If this is accepted 63% had some sort of schema that could be used to successfully answer this type of question and others did not have the required mental constructions relating to APO for this type of question.

Question 2: inverse concept of complex number

Question 2: inverse concept of complex number

7. If $z = x + iy$ then the inverse of z is denoted as z^{-1} is?

a) $\frac{x}{x^2-y^2} + \frac{iy}{x^2-y^2}$
b) $x - iy$
c) $\frac{x}{x^2+y^2} - \frac{iy}{x^2+y^2}$
d) $(x^2 + y^2)(x + iy)$

Table 7. Shows Option Distribution On Question 2 (n = 41)

Question	a	b	c*	d	No Response	Difficulty index	Discriminating index (*100)
2	9	9	18	5	0	43.90	67

Source: field survey

Table 7 shows the option distribution frequency on question 2 as chosen by the students.

The difficulty index of question 2 is about right for an average student (see Table 1) while the discrimination index suggested that there was a very good discrimination among able students from those who were less able to correctly answer this question (see Table 2). This question tested the students' level of understanding on complex number inverse by applying conjugate of complex number as shown in equation 4. It was observed based on the option distribution that some students misinterpreted inverse of complex number as conjugate of complex number and did not remember the concept of inverse. It was also discovered during the interview when students were asked 'Have you heard about inverse and why did you pick your option?'

S1- I know inverse from matrix and we were taught, it should give us identity but 'a' should be the answer.

S2- We had solved this kind of question before, but I did not remember

S5- Yes, b is more understandable, is it not the conjugate?

The students were conversant with the concept of inverse but there was a challenge in its interpretation and application. Students S1,S2 and S5 did not have the required schema to link complex number to inverse property of an algebraic group, but student S1 had good process level mental construction on inverse property of matrices. About 40% of the students had the required schema. Teachers should emphasize on inverse of functions so as prepare students minds on deriving inverse of function, not only in matrices. If this is accepted, the implication is that the majority of the students did not have the required mental constructions relating to APO for this type of question and had some sort of schema that they could use to successfully answer this question.

Question 3. Identity Concept of Complex Number

8. If a complex number $z = x + iy$ is said to have an identity under product operation. Hence, which of the following statements is true, to give the result $x + iy$?

a) $(x + iy)(0 + i)$
b) $(x + iy)(x - iy)$
c) $(x + iy)(1 - i0)$
d) $(x + iy)(1 - iy)$

Table 8: Option distribution and indices for Question 3(n = 41)

Question	a	b	**c***	d	No Response	Difficulty index	Discriminating index (*100)
3	7	11	16	10	2	39.02	50

Source: field survey

Table 8 shows the option distribution frequency on question 8 as chosen by the students.

The difficulty index of question 3 is about right for an average student (see Table 1) while the discrimination index suggested that there was a very good discrimination among able students from those who were less able to correctly answer this question (see Table 2).The concept and application identity are seen from table 11 not to be well understood by majority of the students (68%). The concept is known to them but they could not remember its application. This was also observed in the interview when the students were asked 'what do you think about identity and why did you choose your option?'

S2- Identity is in matrix

S4 – I have heard about it before, I just picked 'd'. I think it should have 1

S5- 'b' is also a complex number, I think it is the right answer

Student S4 noticed that 1(one) might be an identity element but did not know why, this showed there was no in-depth knowledge on the concept. It was observed that S2 and S4 did not have the required schema (not even at action level). However, it was observed that they were conversant with the word identity (since 1 is the identity on operator multiplication on real numbers), which is required for action level of understanding. Note that the choice of S5 had no basis. If this is accepted we could say students did not have the required mental constructions relating to APOS for this type of question.

Question 4. Associative Concept of Complex Number

11. The associativity rule of complex numbers can be described as

a) $x_1(x_2 + x_3) + iy_1(y_2 + y_3)$
b) $(x_1 + x_2 + x_3) - i(y_1 + y_2 + y_3)$
c) $z_1(z_2 + z_3)$
d) $(x_1 + x_2 + x_3) + i(y_1 + y_2 + y_3)$

Table 9: Option Distribution and Indices for Question 4 (n = 41)

Question	a	b	c	**d***	No Response	Difficulty index	Discriminating index (*100)
4	4	7	24	5	2	12.20	17

Source: field survey

Table 9 shows the option distribution frequency on question 4 as chosen by the students.

The difficulty index of question 11 indicates that this question was difficult for the participant (see Table 1) while the discrimination index suggested that there was a very weak discrimination among able students from those who were less able to correctly answer this question (see Table 2). The option distribution confirms that the majority of the students did not understand the difference in the basic concepts of associative and distributive function (majority picked 'c'), it was also observed in the interview when students were asked 'What do you understand by associativity and why did you pick your option?'

S3- If you add 'a' together, you will get it

S6- Since it is an associate question, 'c' should be the answer.

Student S6 knowledge is directed to distribution property and S3 has a wrong but related idea as S6. Hence The students lacked adequate knowledge of associative and distributive properties, even thought the concept was familiar to them, the majority showed low level of understanding of the concept even though they were taught in their first year.. If this is accepted then such students did not have the required mental constructions relating to APO for this type of question about 11% of the students have the require insight schema of the question. The signs that differentiate distribution and association property should be explained so as not to confuse students' in its application.

SUMMARY OF INTERVIEWS

Based on the fact that participates with the lowest marks were invited for the interview so as to concentrate on their weak areas and to consolidate tutoring in similar areas, it was observed that utmost of them showed low position of understanding in the content conception since some of their responses showed that they had forgotten the prerequisite content (properties of abeilan algebraic group),but showed a good position of soberness since they accepted to be canvassed. The students that fell below the average marks were advised to attend tutorial classes to update and upgrade their understanding of the concepts (complex numbers in relation to algebraic group and other related topics).

CONCLUSION

This paper is based on diagnostic test questions for the concept of complex number and in relation to algebraic group concept with the aim of improving students' ability to correctly apply the concept of their prerequisite knowledge on algebraic group to complex numbers and to know student schema on complex numbers. The test question was about right average for students (difficulty index) and adequate in discrimination but the $P_{KR_{20}}$ reliability value, which was 0.38 is considered unacceptable. This was further investigated using APOS theory. The study confirmed that most students experienced difficulties in applying basic concept of algebraic group (prerequisite knowledge) to achieve complex number $(\mathbb{C} - \{0\}, *)$ as shown for questions. Since all the students interviewed acknowledged that they had been taught the concept of commutative, associative, identity and inverse in their first year. It is trivial to see from the diagnostic test and the interview that the failure of most students was based on their inability to remember or correctly apply relevant prerequisite knowledge when and where applicable. That showed their level of insight on the subject matter. The research showed the students' level of insight into the complex number in relation to algebraic abelian group. This could be better if the students' have more knowledge on the importance of prerequisite on mathematical concepts and it is also shown that there is good relation in the interpretation of interviews data through APOS and analysis of option distribution (option picked by students) if the options are designed in such a way.

The concept of commutativity were mostly understood, owing to the percentage of correct scores on the option distribution choice (63% in question 1), next to inverse (44% in question 2), then, identity (39% in question 3). However, associativity properties were mostly mistaken for distribution as shown by option distribution and APOS theory (12% in question 4), hence, the unacceptable reliability (0.38) could be traced to the students mental construction towards attempting the questions as detected by the APOS theory. Teaching based on the APOS generic decomposition should be emphasized upon, and its relation to the relevant prerequisites, so as to create required APO mental construction and schema.

RECOMMENDATIONS

The following recommendations were considered appropriate.

1. Most courses prerequisite might not be indicated in the curriculum but a teacher should extract relevant prerequisite knowledge and skills while reading the curriculum and confirm that the students have adequate knowledge and skills relevant to the areas/concepts before the commencement of the consequential course.
2. Teachers need to lay emphasis on the fact that reading Mathematics course/topic to pass alone without in-depth knowledge or adequate schema is no learning since every Mathematical concept will be a prerequisite at a higher stage.
3. Applicable prerequisite knowledge must be revised with students or tutorials on it should be staged so as to refresh their intellect.
4. Student needs to understand that no Mathematics solving concept, theorems or course is in isolation, any knowledge gained at any stage could applicable at another stage.
5. The first week of every Mathematical course should include review of relevant prerequisite of the course so as to bring the students up the speed, and to avoid teaching in abstract.

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