e-ISSN: 2549-5070
p-ISSN: 2549-8231

Journal of Medives : Journal of Mathematics Education IKIP Veteran Semarang
Volume 7, No. 2, 2023, pp. 175 - 189

https://doi.org/10.31331/medivesveteran.v7i2.2515

Development of Instructional Materials for Introduction to Algebraic Structures: Ring Theory with Guided Discovery

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***Received**: February 2023. **Accepted**: Juny 2023. **Published**: July 2023.*

ABSTRACT

This study presents the development of Teaching Materials for Introduction to Algebraic Structures: Ring Theory with Guided Discovery, addressing the need for comprehensive instructional resources in abstract algebra courses. Following the ADDIE development model (Analysis, Design, Development, Implementation, Evaluation), the research systematically creates and refines the Teaching Materials. The resulting resource includes a title page, carefully crafted learning objectives aligned with the desired Student Learning Outcomes (SLP), and well-organized content arranged into 12 chapters. Each chapter incorporates student learning guides and engaging activities, encouraging active participation and independent exploration. The Teaching Materials undergo rigorous validation, with results indicating a validity level of 88%, meeting the necessary criteria for instructional effectiveness. Based on the evaluation conducted at the final stage of the study, the Teaching Materials demonstrate high practicality, scoring 87%, and are deemed effective with a quality rating of 66%. This study contributes significantly to the field of mathematics education by addressing the need for effective lecture companions in introductory algebraic structures courses. By incorporating guided discovery techniques, the developed Teaching Materials foster increased student activity and knowledge construction. The utilization of the ADDIE development model ensures a systematic and well-structured approach to material creation. The comprehensive organization of content, along with the inclusion of student learning guides and activities, provides a cohesive and interactive learning experience for students. The validation and evaluation results further underscore the efficacy of the Teaching Materials, affirming their validity and practicality in promoting enhanced student learning outcomes. These findings highlight the potential of the Teaching Materials for Introduction to Algebraic Structures: Ring Theory with Guided Discovery to positively impact algebraic structures education and contribute to the pedagogical advancement in mathematics instruction.

***Keywords**: guided discovery, teaching material development, ring theory.*

How to Cite: Kurniawan, P., & Yulianto, A. (2023). Development of Instructional Materials for Introduction to Algebraic Structures: Ring Theory with Guided Discovery. *Journal Of Medives : Journal Of Mathematics Education IKIP Veteran Semarang*, *7*(2), 175 - 189.

INTRODUCTION

Abstract algebra course, is one of the axiomatic mathematics material that is full of definitions and theorems. Studying this material really helps instill logical reasoning, so it is useful in studying other parts of axiomatic mathematics (Isnarto, 2008). The characteristic of an algebraic or abstract algebraic structure is that it has a strict and coherent axiomatic deductive structure, loaded with abstract concepts, both in terms of definitions and theorems (Panggabean, 2015).

According to the observations of the researcher as a lecturer in introductory algebraic structures courses (1 and 2), this course is material that is difficult for students to understand. This is evidenced by the results of routine student exam work, where the average score does not reach half of the maximum score (41%). Dubinsky et al (1994) also explained that most students in abstract algebra classes had great difficulty understanding what the lecturer explained. Arnawa (2009) expressed the same thing, that abstract algebra is a subject that is difficult to learn and difficult to teach. This difficulty arises not only because the lecture material is fairly abstract, but also because students are required to have good deductive and axiomatic proof skills. The ability to abstract concepts and build evidence is the main obstacle for students.

The results of interviews with several students indicated that another obstacle experienced by students was that they did not have sufficient learning resources. Most of the existing teaching materials are English textbooks which are difficult for most students to understand. This happens because not many students master English well. In addition, some of the existing Indonesian language teaching materials are mostly focused on theory and do not focus on the steps in the students' knowledge construction process in proving a theorem. Even though most of the students did not have sufficient abstraction skills, they could not even understand even the initial definitions. Moreover, students are immediately forced to explore several theorems when they do not fully understand the related definitions.

Starting from this problem, researchers see the need for learning resources that can develop abstraction skills and deductive reasoning abilities that can be developed through the activity of writing mathematical proof (Cyr, 2011). The activity of writing mathematical proofs can be developed through the use of appropriate teaching materials and learning methods. Isnarto et al (2014) have researched that the application of Guided Discovery Learning through Motivation to Reasoning and Proving Tasks (GDL-MRP Tasks) in abstract algebra learning has a positive effect on students' ability to construct evidence, understand evidence, and think critically. In addition, Jamilah and Fadillah (2015 & 2017)) have developed algebraic structure teaching materials that can improve students' mathematical proving abilities even without the application of a particular learning model.

Researchers consider that the

development of teaching materials needs to collaborate with the application of certain learning models. Through this research, the Guided Discovery model will be used. This is done so that the developed teaching materials can contain discovery tasks to guide students to be actively and constructively involved (Alfieri et al, 2011). Through this research, ring theory teaching materials with guided discovery will be developed. It is hoped that this teaching material can be used as a reference for the main learning resource that is more effective in assisting students in understanding the material and abstracting abstract algebraic concepts and writing down the evidence needed in developing their deductive reasoning abilities more actively and optimally through a series of discovery tasks.

METHOD

This research is a development research using the ADDIE development model (Analysis, Design, Development, Implementation, Evaluation). The ADDIE model uses a systems approach, namely dividing the learning planning process into several steps, to organize the steps into logical sequences, then using the output of each step as input for the next step (Januszewski & Molenda, 2007). The phases of developing ring theory teaching materials with guided discovery are modified from the development of ADDIE steps according to Gagne, Wager, Golas, and Keller (Januszewski & Molenda, 2007) in Table 1.

Table 1. Development of ADDIE Steps According to Gagne, Wager, Golas, and Keller (Januszewski & Molenda, 2007)

Analysis	a) First determine the needs for which instruction is the solution. b) Conduct an instructional analysis to determine the target cognitive, affective, and motor skill goals for the course. c) Determine what skills the entering learners are expected to have, and which will impact learning in the course. d) Analyze the time available and how much might be accomplished in that period of time. Some authors also recommend an analysis of the context and the resources available.
Design	a) Translate course goals into overall performance outcomes and major objectives for each unit of the course. b) Determining the instructional topics or units to be covered, and how much time will spent on each. c) Sequence the units with regard to the course objectives. d) Flesh out the units of instruction, identifying the major objectives to be achieved during each unit. e) Define lessons and learning activities for each unit. f) Develop specifications for assessment of what students have learned.
Development	a) Make decisions regarding the types of learning activities and materials. b) Prepare draft materials and activities. c) Try out materials and activities with target audience members. d) Revise, refine, and produce materials and activities.
Implementation	a) Market materials for adoption by instructors and potential learners. b) Provide help or support as needed.
Evaluate	a) Implement plans for learners' assessment. b) Implement plans for program evaluation. c) Implement plans for course maintenance and revision.

The module is considered feasible if it meets the following criteria: valid, practical, and effective. In the process of developing teaching materials, validity criteria are reviewed in terms of content, material, format, and language. The validity test was carried out using an expert validity questionnaire which was analyzed descriptively and quantitatively. Teaching materials are said to be valid if they get an average value of 80% of the total score.

The practicality criteria are reviewed in terms of the ease of use of teaching materials which are assessed by using a practicality questionnaire of teaching materials. Teaching materials are said to be practical if they get a maximum score of 3 out of 5, or achieve a minimum score of 60% of the total score.

RESULTS AND DISCUSSION

Analysis

Based on the results of the analysis, it was found that ring theory teaching materials were needed which contained constructive study guides for students. The main learning outcomes that will be achieved and more emphasis on in the development process are the achievements in the domain of specific knowledge and skills, namely: mastering the theoretical concepts of mathematics including mathematical logic, discrete mathematics, algebra, analysis, and geometry, as well as a theory of probability and statistics in depth; and able to develop mathematical thinking, starting from procedural or computational understanding to a broad understanding including exploration, logical reasoning, generalization, abstraction, and formal proof. It is hoped that this teaching material will later have an influence on these two achievements, namely in mastering algebraic theoretical concepts and developing mathematical thinking including logical reasoning, generalization, abstraction, and formal proof.

Design

At the design stage, the first step is to design teaching materials. Some of the components that will be arranged in the ring theory module with guided discovery are the title page, learning objectives that are in accordance with the lesson plan, material arranged in several chapters, guided questions, and practice questions.

Development

In this step, the activity carried out is the realization of the design. The module has been prepared according to the results at the design stage. The components that have been listed in the module are material, practice questions, and several student activity sheets in the form of activities that are guided through a series of questions. The realization of the design stage includes a series of materials and guiding questions which are packaged into 8 chapters (Figure 1).

After the module is equipped with its components, content validation will be carried out by the validator. The purpose of this validation is to determine the validity and feasibility of the developed module. The validation results from material experts and media experts show that the developed module is valid and can be used with revisions.


Daftar Isi

Figure 1. Table of Contents

The next step is creating teaching material validity instruments which include module content validation sheets and module practicality sheets, both of which are in the form of a questionnaire using a Likert scale. Each of the available statements has five possible answer choices which are an

assessment of the validity and practicality of the ring theory module with guided discovery.

There are 4 aspects were going to assess, they were Content Eligibility, Eligibility of Presentation, Language Assessment, and Guided Discovery Learning. Validity Instrument Assessment Points for Material Experts on Content Adequacy Aspects are:

1. Material completeness.
2. Breadth of material.
3. The depth of the material.
4. Accuracy of concepts and definitions.
5. Accuracy principle.
6. Accuracy of facts and data.
7. Sample accuracy.
8. Accuracy of questions.
9. Accuracy of drawings, diagrams, and illustrations.
10. Accuracy of notations, symbols, and icons.
11. Accuracy of reference literature.
12. Reasoning.
13. Linkage.
14. Communication (write and talk).
15. Application.
16. Material attractiveness.
17. Encourage to seek further information
18. The suitability of the material with the development of science.
19. Draw actual diagrams and illustrations.
20. Library updates.

Validation Instrument Assessment Points for Material Experts on Presentation Adequacy Aspects are:

1. Systematic consistency of presentation in activities.
2. Presentation order.
3. Examples of questions in each learning activity.
4. Practice questions at the end of each learning activity.
5. Answer keys or work guides on practice questions
6. Feedback about practice
7. Introduction
8. Glossary or index
9. Bibliography
10. Summary
11. Student involvement
12. Introductory section
13. Content section
14. Concluding Section

Validity Instrument Assessment Items for Material Experts on Language Assessment Aspects are:

1. Sentence structure accuracy
2. Sentence effectiveness
3. Standardization of terminology
4. Message readability
5. Language rule accuracy
6. Ability to engage and motivate learners
7. Promotion of critical thinking skills
8. Intellectual suitability for students
9. Alignment with students' emotional development levels
10. Consistency and integration across learning activities
11. Coherence and consistency between paragraphs
12. Consistent use of terminology
13. Consistent use of symbols

Validity Instrument Assessment Points for Material Experts on Guided Discovery Learning Aspects are:

1. The module provides a stimulus or motivation to students at the

beginning of each material.
2. The module supports problem identification by students so as to create learning situations that involve students learning actively.
3. The module provides opportunities for students to collect and process data or examples or symptoms that occur in specific contexts.
4. The module provides opportunities for students with the guidance of lecturers to verify the proposed temporary concept or theory.
5. The module provides a forum for generalization through the process of drawing conclusions from observations.
6. The module makes the teacher a facilitator and guide.
7. Students can independently find a concept or theory.

Item Validity Instrument Sheet for Media Experts are:
1. The physical size of the module
2. Module cover layout
3. Layout consistency
4. Harmonious layout elements
5. Complete layout elements
6. Layout accelerates understanding
7. understanding.
8. Simple Book Fill Typography
9. Easy to Read Typography
10. Typography of Book Contents Makes it Easy to Understand
11. Fill illustration

Item Validity Instrument Sheet for Student Responses includes Appearance, Presentation of material, and Benefit aspect.

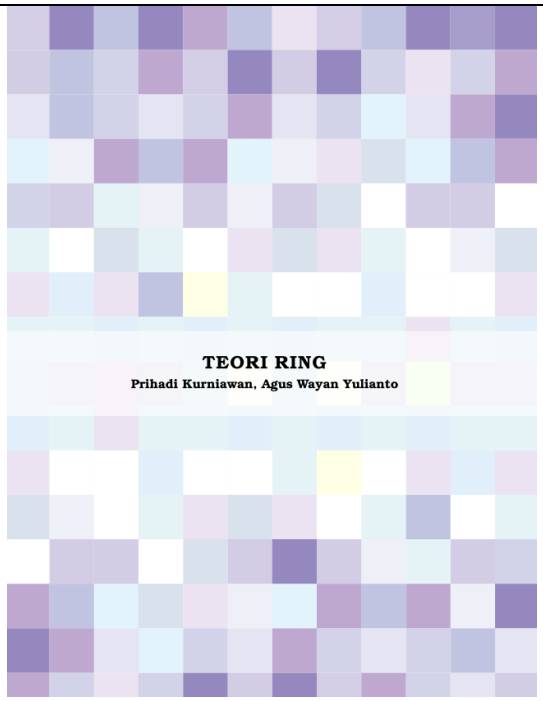
The result of validity assessment is shown on Table 2.

Table 2. Validity Assessment

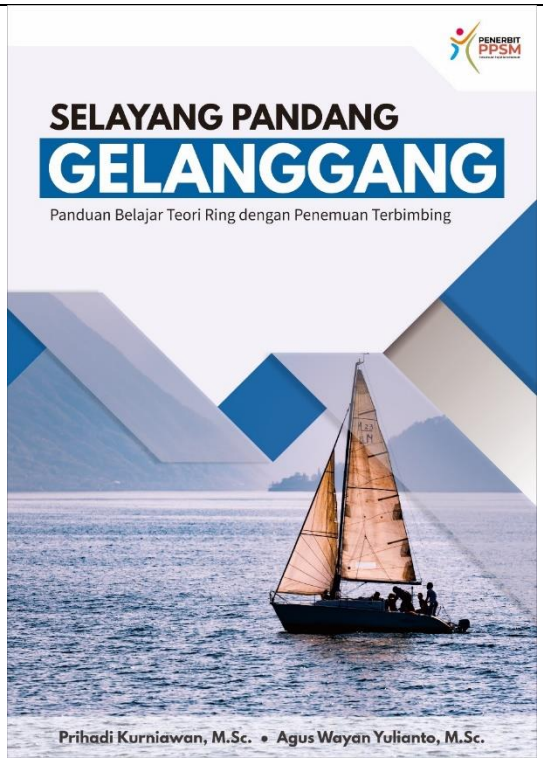
No	Criteria	% Mean on each aspects			% Mean	Criteria
		I	II	III		
	MATERI					
1	Aspects of content eligibility	87%	81%	93%	87%	Very Valid
2	Eligibility aspects of presentation	79%	77%	94%	83%	Valid
3	Aspects of language assessment	89%	86%	94%	90%	Very Valid
4	Guided discovery learning aspect	94%	83%	83%	87%	Very Valid
	MEDIA					
5	Graphics qualification	90%	85%	95%	90%	Very Valid
	Mean				88%	Very Valid

The validation results are then followed up with revisions. Revision of the module in terms of material and media was carried out by taking into account the validator's suggestions. The results of the revision can be seen in Table 3.

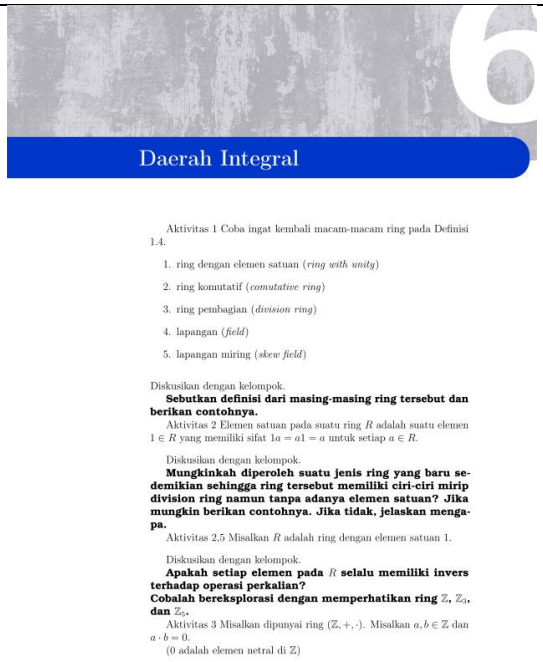
Table 3. Revision of Media



Cover before revision



Cover after revision



6

Daerah Integral

Aktivitas 1 Coba ingat kembali macam-macam ring pada Definisi 1.4.

1. ring dengan elemen satuan (*ring with unity*)
2. ring komutatif (*comutative ring*)
3. ring pembagian (*division ring*)
4. lapangan (*field*)
5. lapangan miring (*skew field*)

Diskusikan dengan kelompok.

Sebutkan definisi dari masing-masing ring tersebut dan berikan contohnya.

Aktivitas 2 Elemen satuan pada suatu ring R adalah suatu elemen $1 \in R$ yang memiliki sifat $1a = a1 = a$ untuk setiap $a \in R$.

Diskusikan dengan kelompok.

Mungkinkah diperoleh suatu jenis ring yang baru sedemikian sehingga ring tersebut memiliki ciri-ciri mirip division ring namun tanpa adanya elemen satuan? Jika mungkin berikan contohnya. Jika tidak, jelaskan mengapa.

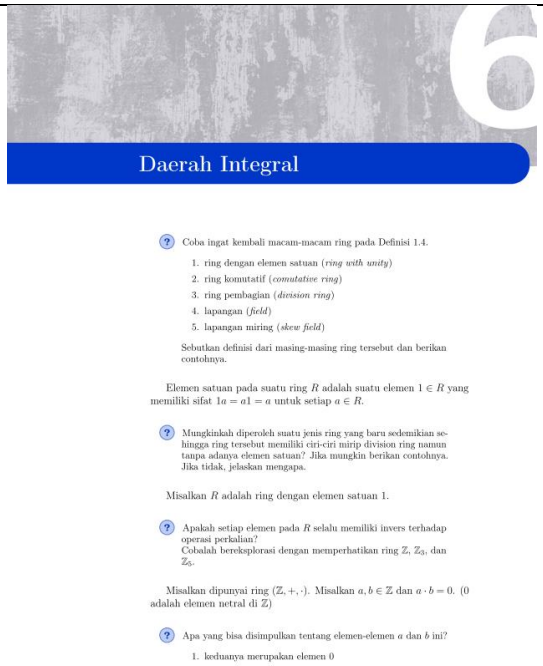
Aktivitas 2,5 Misalkan R adalah ring dengan elemen satuan 1.

Diskusikan dengan kelompok.

Apakah setiap elemen pada R selalu memiliki invers terhadap operasi perkalian?
Cobalah bereksplorasi dengan memperhatikan ring $\mathbb{Z}$, $\mathbb{Z}_3$, dan $\mathbb{Z}_5$.

Aktivitas 3 Misalkan dipunyai ring $(\mathbb{Z}, +, \cdot)$. Misalkan $a, b \in \mathbb{Z}$ dan $a \cdot b = 0$.
(0 adalah elemen netral di $\mathbb{Z}$)

Page setting before revision



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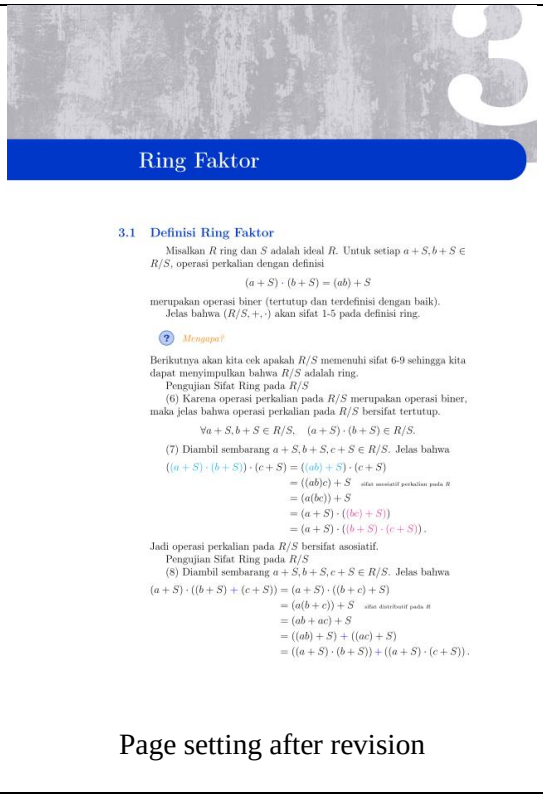
Misalkan dipunyai ring $(\mathbb{Z}, +, \cdot)$. Misalkan $a, b \in \mathbb{Z}$ dan $a \cdot b = 0$. (0 adalah elemen netral di $\mathbb{Z}$)

? Apa yang bisa disimpulkan tentang elemen-elemen a dan b ini?

1. keduanya merupakan elemen 0

Page setting after revision



3

Ring Faktor

3.1 Definisi Ring Faktor

Misalkan R ring dan S adalah ideal R. Untuk setiap $a+S, b+S \in R/S$, operasi perkalian dengan definisi

$$(a+S)\cdot(b+S) = (ab)+S$$

merupakan operasi biner (tertutup dan terdefinisi dengan baik).

Jelas bahwa $(R/S, +, \cdot)$ akan sifat 1-5 pada definisi ring.

Mengapa?

Berikutnya akan kita cek apakah R/S memenuhi sifat 6-9 sehingga kita dapat menyimpulkan bahwa R/S adalah ring.

Pengujian Sifat Ring pada R/S

(6) Karena operasi perkalian pada R/S merupakan operasi biner, maka jelas bahwa operasi perkalian pada R/S bersifat tertutup.

$$\forall a+S, b+S \in R/S, \quad (a+S)\cdot(b+S) \in R/S.$$

(7) Diambil sembarang $a+S, b+S, c+S \in R/S$. Jelas bahwa

$$\begin{aligned}((a+S)\cdot(b+S))\cdot(c+S) &= ((ab)+S)\cdot(c+S)\\ &= ((ab)c)+S \quad \text{sifat asosiatif perkalian pada } R\\ &= (a(bc))+S\\ &= (a+S)\cdot((bc)+S)\\ &= (a+S)\cdot((b+S)\cdot(c+S)).\end{aligned}$$

Jadi operasi perkalian pada R/S bersifat asosiatif.

Pengujian Sifat Ring pada R/S

(8) Diambil sembarang $a+S, b+S, c+S \in R/S$. Jelas bahwa

$$\begin{aligned}(a+S)\cdot((b+S)+(c+S)) &= (a+S)\cdot((b+c)+S)\\ &= (a(b+c))+S \quad \text{sifat distributif pada } R\\ &= (ab+ac)+S\\ &= ((ab)+S)+((ac)+S)\\ &= ((a+S)\cdot(b+S))+((a+S)\cdot(c+S)).\end{aligned}$$

Page setting after revision

Implementation

At the implementation stage, the activities carried out are limited trials whose implementation description is as follows. At this stage, a module trial was carried out that had been validated and revised. The researcher tested the module on a small group (limited test). This trial is intended to see the level of practicality of the module.

The trial was carried out in one class which had an introductory algebraic structure course 1, namely in the MAT-7A class which consisted of 9 students. The implementation was carried out at a meeting that discussed the material for the definition of ring and subring. Students are asked to observe the learning of Chapter 1.

Lecturers and students discuss and conduct questions and answers to discuss difficulties related to the implementation of group discussions. Then the lecturer provides feedback.

Evaluation

After the students used the module, the researcher distributed questionnaires. This questionnaire aims to see student responses to the modules that have been developed in terms of several aspects that meet the practicality criteria.

The practicality of the developed module was analyzed based on the results of the student response questionnaire. The practicality of the module is reviewed from the aspect of appearance, presentation of material, and benefits. Recapitulation of the percentage of each statement and the average of each questionnaire criteria for student responses to the use of the module grouped based on practicality criteria is presented in Table 4.

It has been found that, in the display aspect, a percentage of 90% is obtained, in the presentation aspect of the material, a percentage is obtained by 82%, and in the benefit aspect, a percentage is obtained by 88%. From these three aspects, an average score of 87% was obtained. Therefore it can be concluded that the modules developed are very practical for use by students.

In accordance with the results of the competency test at the small-scale trial meeting, namely, after students used the ring theory module-assisted lecture scheme with guided discovery it was found that many students scored below 60 out of 9 students as many as 3 students. This means that the percentage level of effectiveness is 66%. According to the criteria, the module is included in the effective category.

CONCLUSION

Based on the research results in the form of the development of ring theory modules with guided discovery and data analysis, it can be concluded as follows.

1. The modules have been prepared based on the flow of the ADDIE development model which includes the Analysis, Design, Development, Implementation, and Evaluation stages.
2. The analysis phase that has been carried out includes identifying and formulating problems, analyzing the need to develop teaching materials, needs analysis,

curriculum analysis, student character analysis, and learning method analysis. The results obtained through the results of student responses through questionnaires are that students need learning modules in the introductory algebraic structure 2 course which emphasizes student activity. Students need a lecture scheme that helps construct knowledge with questions so as to increase student activity. The alternative learning method used in this research is guided discovery. This method is embedded in the module.

3. The design stage that has been carried out includes designing teaching materials in the form of ring theory modules with guided discovery which contain the title page, learning objectives that are in accordance with the SLP, material arranged in 9 chapters, student activities contained in the module, and practice questions. After the design is complete, it is continued with the selection of library sources and teaching material validity instruments.
4. The development stage carried out is the realization of the design so that a ring theory module with guided discovery is obtained which is in accordance with the results at the design stage. After this realization is complete, it is followed by module validation.
5. Based on the validation results by three material expert validators and media experts, it was found that the validity of the ring theory module with guided discovery has a validity level of 88% which is included in the valid criteria and can be used with revision. Revisions are then carried out before the next stage is implemented.
6. The implementation stage of the small-scale module for students of the 7th semester mathematics study program received good responses and responses.
7. At the evaluation stage, the evaluation is carried out summatively using a student response questionnaire and competency test scores. Based on the questionnaire data, it was found that the practicality criteria for the ring theory module with guided discovery were at a value of 87% which was included in the very practical criteria. Then the module is tested for its effectiveness. It was found that the level of effectiveness of the module in students of the 7th semester mathematics study program using classical criteria was included in the effective criteria with a quality of 67%.

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