

Analysis of Students' Mathematical Reasoning in Solving Pythagorean Theorem Problems

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ABSTRACT

The purpose of this research is to describe students' understanding of the concept of the Pythagorean theorem. This research was carried out at Muhammadiyah 7 Bayat Middle School. This research uses descriptive qualitative research methods. The data collection technique is written test questions. The subjects of this research consisted of 20 class VIII students. The data analysis techniques used are data reduction, data presentation, and conclusions. The results of this research were that 45% of students were able to solve questions on applying questions (C3), in the good category 25%, and 30% in the fair category. In the analysis questions (C4), 50% were in the very good category, 30% in the good category, and 20% in the fair category. In the evaluation questions (C5), 25% were in the very good category, 45% in the good category, and 30% in the fair category.

Keywords: mathematical reasoning, the concept of understanding, Pythagorean theorem.

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INTRODUCTION

Mathematics learning is a learning activity in understanding the meaning and relationship of symbols and then applying them in real situations. According to (Nurhikmayati, 2019) activities in mathematics learning can encourage students to develop the ability to understand situations and find solutions to the problems they face. Learning mathematics not only develops cognitive abilities in the sense of being good at calculating, but learning mathematics can develop several aspects, including cognitive, affective and psychomotor aspects. Apart from that, in the field of education, students need the ability to think logically, analytically, critically, and be able to work together in all learning, especially in learning mathematics (Lesi & Nuraeni, 2021). From the explanations above, it can be concluded that education is a conscious effort needed by every human being to achieve the ability to think logically.

According to Johnson and Rising (Rahmah, 2018) mathematics is a pattern of thinking, a pattern of organizing, logical proof, mathematics is a language that uses terms that are defined carefully, clearly and accurately represented with symbols and conciseness, more in the form of symbolic language regarding ideas rather than sounds. According to (Borrego, 2021) Mathematics is knowledge in an organized structure, where the properties of a theory are determined deductively based on undefined elements, axioms, properties or theories whose truth and harmony are proven. Therefore, students must master mathematical knowledge from an early age because mathematics is one of the most important subjects in school which can increase students' knowledge in thinking critically, logically and

rationally as well as carefully, effectively and efficiently.

Reasoning is one of the basic competencies in mathematics. According to Ismawati Waliulu et al, (2022), reasoning is a thought process that attempts to connect known facts or evidence towards a conclusion. In other words, reasoning is a cognitive process in the form of drawing conclusions from arguments (premises) that are considered valid. Reasoning is also a mental process that develops the mind from several facts and principles. Turmudi in (Santosa et al., 2020) states that mathematical reasoning is a brain routine or habit like other habits that must be developed consistently using various contexts. So mathematical reasoning has a very important role in a person's thinking process. This is reinforced by (Kusumawardani et al., 2018) Reasoning is also a foundation in learning for students to improve students' mathematical understanding and reasoning by providing appropriate learning. Mathematics learning aims to develop students' abilities to obtain maximum results, therefore mathematical reasoning abilities must always be familiarized and developed.

The reasoning aspect is an important aspect that individuals must have in learning mathematics (Awaliyah & Fitrianna, 2018). According to Romadhina (Aprilianti & Zanthi, 2019) who refers to the Technical Guidelines for Regulations of the Director General of Basic Education, Ministry of National Education Number 506/C/Kep/PP/2004, detailing the aspects of indicators of mathematical reasoning ability as follows; a) Raise allegations; b) Carry out mathematical manipulation; c) Drawing conclusions, compiling evidence, providing reasons or evidence for the correctness of the solution; d) Checking the validity of an

argument; and e) Drawing conclusions from the statement. Therefore, these indicators are used as a reference in this research. One of the problems in mathematics that requires students' mathematical reasoning to solve is geometry. Geometry is studied all levels of education, from low to high levels or from elementary, middle and high school, even to university. Geometry in learning includes geometric shapes (flat planes and geometric shapes), lines and angles, similarity, congruence, transformation, and analytical geometry (Asih & Imami, 2021). The concepts of geometry that have been studied since elementary school have not yet been mastered by the majority of students. This will have an impact on students at junior high school level, students will still not have mastered the concepts of geometry. One of them is the Pythagorean theorem. According to (Rohmah, 2020) the Pythagorean theorem is one of the materials that must be studied in class VII, this material is very closely related to the previous materials, therefore the concepts of the Pythagorean theorem must be mastered by students, otherwise students will experience difficulties in solving problems. The Pythagorean theorem, also known as the Pythagorean theorem, is a theorem that shows the relationship of the sides of a right triangle. Mastery of the concepts in the Pythagorean theorem material needs to be improved and must be understood by every student.

Concept understanding consists of two main words, namely understanding and concept. According to Sudjiono in (Fadlilah, 2015), understanding is a person's ability to understand something after it is known and remembered, while a concept is the result of a person or group's thinking which is expressed with a definition so that a knowledge

product is created including principles. Understanding mathematical concepts is a student's ability to explain the relationship between concepts and apply concepts or algorithms flexibly and accurately to solve various problems by understanding existing mathematical concepts (Febriyani et al., 2022). Understanding concepts is one of the important factors that students must have as a result of the learning process, this ability can be developed in mathematics learning (Azizah et al., 2021). In response to this, the researcher felt it necessary to conduct research to investigate the extent of understanding students' concepts solving problems at Muhammadiyah 7 Bayat Middle School students in class VIII.

METHOD

This research uses descriptive qualitative research methods. Descriptive qualitative research is research that has the aim of describing completely and in depth the events of the various phenomena being studied. researchers describe students in solving problems (Fadli, 2021). The subjects in this research were 20 class VIII students of SMP Muhammadiyah 7 Bayat who had studied the Pythagorean theorem material.

The main instrument in this research is a description and documentation test related to the Pythagorean theorem material. Students work on mathematical reasoning test questions consisting of three description test questions based on Bloom's taxonomy, namely applying, analyzing and evaluating. The instruments that have been prepared are then tested to find out whether the questions are suitable for use or not. The questions are given to students who have studied the Pythagorean theorem material. After

the researchers used the help of Excel and the technique used in testing was

the Pearson Correlation technique (Irsyad & Fauzi, 2021). Results Table 1;

Table 1. Pearson validity test

r-count	r-table	Criteria
0,631	0,514	Valid
0,573	0,514	Valid
0,669	0,514	Valid

Data analysis is carried out periodically during the research process, this is done so that the data obtained is more systematic and thus prevents data errors. According to Miles & Huberman (Suryadi, 2019), qualitative data analysis techniques generally have three stages, including 1) Data reduction stage, in this stage the researcher carries out an analysis of the data by analyzing the results of students' answers to determine the students' stages in answering description questions. 2) At the data presentation stage, in this stage the researcher presents the results of the

analysis and presents them in the form of narrative text or the form of short descriptions without any manipulation which will then be analyzed according to the main problem and conclusions will be drawn to answer the problems in the research in this study. 3) At the conclusion stage, at this stage the researcher draws conclusions that have been obtained from the data reduction and data presentation stages. Researchers categorize students' understanding of mathematical reasoning concepts using modified criteria (Aminah, 2021), as in Table 2

Table 2. Category of understanding mathematical reasoning concepts

Range	Category
67-100	Very good
34-66	Good
0-33	Enough

RESULTS AND DISCUSSION

The results of the research are the results of students' work on test instruments regarding students' understanding of mathematical reasoning concepts in the Pythagorean

theorem material. A descriptive description of the research results is presented in Table 3.

Table 3. Description of research results

Category	Applying (C3)		Analyzing (C4)		Evaluating (C5)	
	The number of students	%	The number of students	%	The number of students	%
Very good	9	45	10	50	5	25
Good	5	25	6	30	6	30
Enough	6	30	4	20	6	30
total	20	100	20	100	20	100

Based on Table 2, There are 9 students, or 45% of students'

understanding of the concept of mathematical reasoning in applying questions (C3). In the good category there are 5 students or 25%, and in the fair category, there are 6 students or 30%. In the analyzing questions (C4) in the very good category, there were 10 students or 50%, in the good category there were 6 students or 30%, and in the fair category there were 4 students or 20%. In the evaluation question (C5), in the very good category, there were 5 students or 25%, in the good category there were 6 students or 30%, and in the fair category there were 6 students or 30%. To deepen the research results, researchers analyzed students' understanding of mathematical reasoning concepts based on Bloom's taxonomy levels.

1. Very Good Category

The following are the student test results in the very good category,

① Diketahui :
 Tinggi nita = 150 cm \rightarrow 1,5 m
 Jarak nita ke bendera = 12 m
 Kepala nita dan tiang = 13 m
 Ditanya :
 tinggi tiang bendera ?
 Dijawab :
 $a^2 = c^2 - b^2$
 $a^2 = 13^2 - 12^2$
 $a^2 = 169 - 144$
 $a^2 = 25$
 $a = \sqrt{25}$
 $a = 5 \text{ m}$
 Tinggi tiang
 Jarak kepala + Tinggi nita
 $= 5 \text{ m} + 1,5 \text{ m}$
 $= 6,5 \text{ m}$

Figure 1. Results Very Good Category

Figure 1 shows that students can solve problems related to question number one, this is proposed using the Pythagorean theorem. For example, students change the subject's height from 150 cm to 1.5 meters, the subject to the flagpole is 12 meters and the distance between the subject's head and the flagpole is 13 meters. students can use the Pythagorean theorem formula in the problem, namely $a^2 = c^2 - b^2$ then

DISCUSSION

The results of student work are described in each question based on Bloom's taxonomy.

Cognitive Level C3 (Applying)

This question requires students to use the Pythagorean theorem formula. Students are asked to determine the height of the flagpole. This question is included in cognitive level C3 with operational verbs (KKO) applied. This C3 cognitive question is given to students in question number one. Students are asked to calculate using procedures or steps to complete the Pythagorean theorem formula. The results of student answers can be categorized into 3 categories, as follows:

students can write numbers into the formula, and the student's answer is correct. This agrees with Sutinah in (Irmayanti et al.,2020) which states that subjects with very good criteria can estimate the completion process correctly. Students can also provide the final result, namely that students can add up the head distance and the height of a person named Nita.

2. Category Good

The following are the student test results in the good category;

① Diketahui : Tita : 150 cm
 $b = 12 \text{ m}$
 $c = 13 \text{ m}$
 Ditanya : $a = ?$
 Jawab : $a^2 = c^2 - b^2$
 $a^2 = 13^2 - 12^2$
 $a^2 = 169 - 144$
 $a^2 = 25$
 $a = \sqrt{25}$
 $a = 5 \text{ m}$

Tinggi tiang
 Jarak kepala + Tinggi nita
 $= 13 \text{ m} + 1.5 \text{ m}$
 $= 14.5 \text{ m}$

Figure 2. The student's answer results are in a good category;

Figure 2 shows that students can solve problems related to question number one, this is shown by the example that the subject's height is 150 cm. The subject is 12 meters from the flagpole and the distance between the subject's head and the flagpole is 13 meters. However, in the process of solving the questions, students made errors due to not being careful in understanding the questions, namely the

students did not convert the child's height into meters. Students can solve problems by using the Pythagorean theorem formula in the problem, namely $a^2 = c^2 - b^2$ then students can write numbers into the formula. Students can get results from solving problems, but the results obtained from students are not accurate. Mistakes that students might make are when entering numbers into the given formula.

3. Category Enough

The following are the student test results in the sufficient category,

1. $c^2 = a^2 + b^2$
 $= 13^2 + 12^2$
 $= 169 + 144$
 $= 313$

Figure 3. The students' answers were in the sufficient category

Figure 3 shows that students are unable to solve problems according to the solution steps. Students do not write down what they know in the questions. Students immediately write answers without providing steps to solve the problem. Students write the formula for

the Pythagorean theorem, but students also misunderstand the problem, students should use the formula by subtracting, students adding instead. So, it can be concluded that students are not able to solve the questions according to the solution steps.

Cognitive Level of Analyzing (C4)

In this question, students are asked to draw a sketch and determine the shortest distance the fisherman can travel to return to place A. This question falls into the cognitive level of Analyzing (C4) with the verb to describe, in the first indicator Proposing a guess, students can write down whatever they know. on the question. Second indicator: Students can manipulate mathematics, students

describe triangle sketches, and solve problems using the Pythagorean theorem formula. For the third indicator, in compiling evidence for truth, students can sketch the form of a right triangle and solve the problem in the question using the Pythagorean theorem formula. The results of student answers can be categorized into 3 categories, as follows:

1. Category Very Good

The following are the student test results in the very good category,

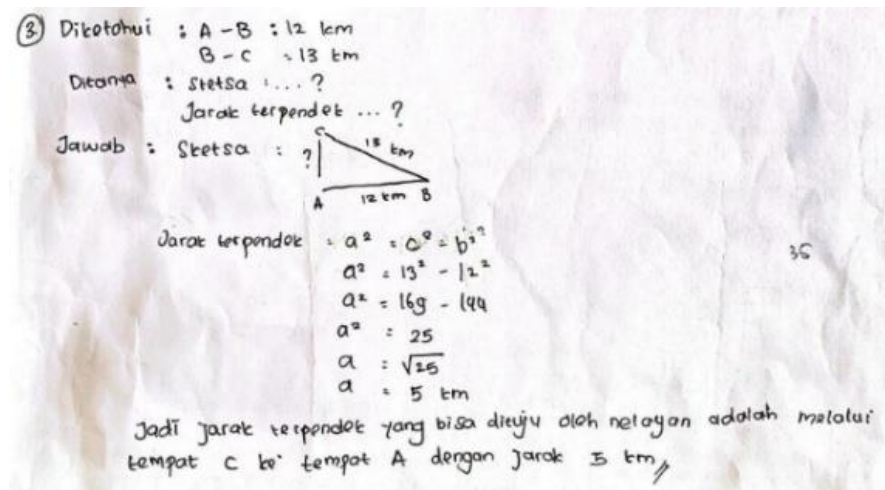


Figure 4. The results of the students were in the very good category

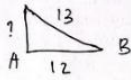
Figure 4 shows that students can solve problems related to question number three, this is proposed by writing down what is known, namely points A to B are 12 km, and points B to C are 13 km. Students can write down what is asked in the question, namely what is the shortest distance traversed by fishermen, and draw a sketch. Students can sketch a right triangle and

students are also able to write answers using the Pythagorean theorem formula accurately and correctly, namely the shortest distance that can be traversed by fishermen from place C to place A is 5 km which according to the opinion (Anisah et al., 2011) which states that students are in the very good category because they have good mathematical reasoning skills.

2. Category Good

The following are the student test results in the good category;

③ Diketahui : A ke B = 12 km
B ke C = 13 km
Ditanya : Jarak terpendek?
Sketsa



$$AC^2 = CB^2 - AB^2$$

$$AC^2 = 13^2 - 12^2$$

$$AC^2 = 169 - 144$$

$$AC^2 = 25$$

$$AC = \sqrt{25}$$

$$AC = 5 \text{ km.}$$

Figure 5. The students' answers were in the good category

Figure 5 shows that students can solve problems related to question number three, this is proposed by writing down what is known, namely points A to B are 12 km, and points B to C are 13 km. Students can write down what is asked in the question, namely what is the shortest distance traversed

by fishermen, and draw a sketch. students can sketch a right triangle and students are also able to write answers using the Pythagorean theorem formula accurately and correctly, namely the shortest distance that can be traversed by fishermen from place c to place a is 5 km.

3. Category enough

The following are the student test results in the sufficient category;

③ $a^2 = c^2 - b^2$
 $= 13^2 - 12^2$
 $= 169 - 144$
 $= \sqrt{25}$
 $= 5$

Figure 6. The results of the student's answers were in the very good category

Figure 6 shows that students are unable to solve the problem according to the solution steps. Students do not write down what is known in the problem. Students write answers without providing steps to solve the

problem. Students write the Pythagorean theorem formula, namely $a^2 = c^2 - b^2$. So students have not been able to solve the problem according to the solution steps.

Cognitive Level C5 (Evaluating)

In this question, **students** are asked to make a decision and provide

reasons for the truth of the question. This question falls into the cognitive

level of Evaluating (C5) with the verb checking, namely students are asked to answer yes or no then students give

reasons for the truth. The results of student answers can be categorized into 3 categories, as follows:

1. Category Very Good

The following are the student test results in the very good category

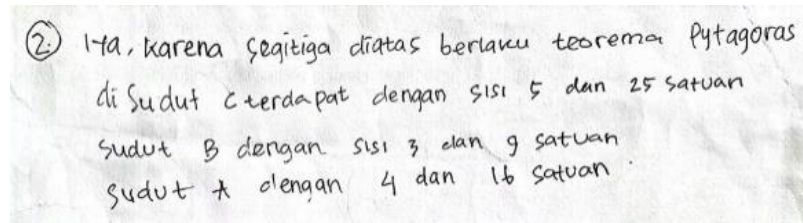


Figure 7. The results of the answer category are very good

Figure 7 shows that students can solve questions according to the indicators. Students can provide answers and students can provide reasons or evidence for truth. Based on the ability to make conjectures, students can give a yes answer. In the ability to compile proof of truth, students can

provide reasons for the question, and the Pythagorean theorem applies. Students also write proof that angle c has 5 sides or 25 units, angle b has 3 sides or 9 units, and angle a has 4 sides or 16 units. So it can be concluded that students can answer questions with solution steps that are by the indicators.

2. Category Good

The following are the student test results in the good category;

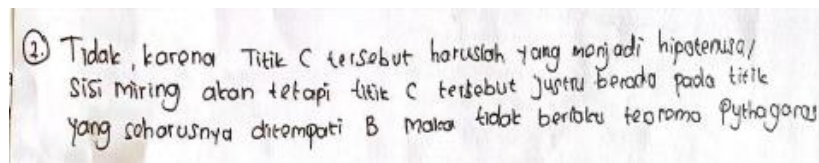


Figure 8. The answer results are in the good category

Figure 8 shows that students can provide answers and students are able to provide reasons or evidence for truth. In ability to make assumptions, students can give a no answer even if the student's answer is not correct. In the indicator of providing reasons or evidence for the truth, students are able

to provide reasons that point c should be the hypotenuse, but if the point is at point b, the Pythagorean theorem does not apply. So it can be concluded that students in the good category are able to answer questions according to the indicators but the students' answers are not correct.

3. Category enough

Student test results in the sufficient category;

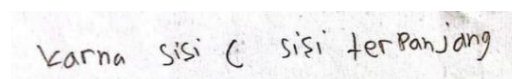


Figure 9. The following are the student test results in the sufficient category;

Figure 9 shows that students are unable to solve the problems in the questions. On the indicator of raising an allegation, the student cannot provide an answer because the student did not answer yes or no. In the indicator of providing reasons or evidence for the truth, students can provide reasons because side c is the longest side. So it can be concluded that students only answer questions with one indicator, namely students only provide reasons or evidence of the truth.

CONCLUSION

Based on the results and discussion in this research, it can be concluded that students' mathematical reasoning abilities in understanding the concept of the Pythagorean theorem in class VIII of SMP Muhammadiyah 7 Bayat in solving Pythagorean theorem problems, at the cognitive level of applying (C3), students are in the very good category, students are able solving steps contained in the problem and answering the Pythagorean theorem formula correctly. In the good category, students can write down the steps to solve the problem but there are still errors. In the sufficient category, students do not meet the criteria because students do not understand the existing problem, students cannot write down the steps to solve the problem, and students immediately write the answer. For the cognitive analysis level (C4) in the very good category, students can sketch a picture of a right triangle. Students are also able to write down the solution steps using the Pythagorean theorem formula. In the good category, students can sketch a right triangle and students can write down the solution steps using the Pythagoras theorem formula, but the student's final answer is not correct. In the sufficient category,

students cannot write down the solution steps. Students just write the Pythagorean theorem formula directly. For the cognitive evaluation level (C5), in the very good category, students can answer questions according to mathematical reasoning indicators correctly, students can answer and students can provide reasons for proof of truth. In the good category, students can answer and provide reasons for the evidence of truth, but the student's answer is not correct. In the sufficient category, students cannot provide answers, but students can provide reasons for the evidence.

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