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Mathematical Language Errors and Academic Performance in Preservice Calculus Teachers

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ABSTRACT

This study explores the impact of mathematical language errors (MLE) on the academic achievement of preservice calculus teachers, addressing the critical issue of how these errors impede mathematical understanding and performance. Conducted within the context of South Africa's education system, where effective mathematics instruction is vital for educational and economic development, this study aims to identify common mathematical language pitfalls, analyze their consequences, and propose strategies to mitigate these errors. An exploratory sequential mixed-methods design was employed, combining qualitative and quantitative approaches to understand the issue comprehensively. The study involved 120 preservice teachers divided into first and second-year cohorts. Data collection included qualitative analysis of student responses through thematic coding and quantitative analysis of assessment results using descriptive statistics, chi-square test, and ANOVA. Significant findings identified two major MLE types: incorrect terminology and ambiguous language errors, which were prevalent and significantly impacted academic performance. First-year students exhibited higher frequencies of these errors, correlating with lower academic scores than second-year students. Statistical analyses confirmed significant differences in error distribution and their impact on performance, highlighting the need for early and targeted interventions. The study contributes to the literature on mathematics education by highlighting the importance of mathematical language in education and provides evidence-based strategies for improving mathematical instruction. By addressing mathematical language errors through explicit instruction, interactive activities, and peer feedback, mathematics educators can enhance students' understanding and performance, ultimately fostering a more effective and inclusive learning environment. Further research is needed to explore the long-term impact of these interventions on teaching efficacy and student outcomes.
Keywords: *Mathematical language errors, Preservice calculus teachers, Incorrect terminology, Mathematical symbols, and Ambiguous language.*

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INTRODUCTION

In South Africa, where mathematics is essential for education and economic development, the importance of effective teaching practices cannot be overstated (Prince & Frith, 2020; Venkat & Askew, 2021). Mathematics is a unique language that people use to communicate, solve problems, engage in recreation, and create art and mechanical tools (Khalo et al., 2022). Mathematical language comprises words, numerals, and symbols, which can be interrelated and independent (Prince & Frith, 2020). However, mathematical language errors (MLE) pose a significant challenge to fostering meaningful mathematical communication (Prinsloo & Harvey, 2020; de Mooij et al., 2022; Тонких, 2023).

Mathematical language errors are defined as errors associated with mathematical symbols and vocabulary (Baidoo, 2019). Makonye (2012) referred to these errors as arbitrary and noted they are a significant cause of learners' mistakes due to their failure to decode and understand mathematical tasks. These errors can occur when learners misinterpret symbols, such as changing a multiplication sign to an addition sign or confusing the numerator with the denominator. Such misunderstandings create barriers to comprehending mathematical concepts (Prince & Frith, 2020). This paper explores the research question: What is the relationship between mathematical language errors and academic performance among preservice calculus teachers, and what are the lived experiences of these preservice teachers in navigating mathematical language challenges? This study aims to bridge the gap between mathematical language and effective mathematical instruction among preservice teachers by identifying common mathematical language errors and suggesting strategies to minimise misconceptions.

Literature Review and theoretical framework

Mathematics education transcends conveying content knowledge; it shapes cognitive development and analytical thinking, essential for success in various fields (Makonye & Moodley, 2023; Naidoo, 2022). Effective education depends on content and the sociocultural context in which it is situated (Taylor, 2021). Sociocultural learning theory provides a valuable lens through which to understand the role of language and communication in mathematical learning (Johnson, 2022). Mathematical language errors include inaccuracies related to mathematical symbols and vocabulary (Baidoo, 2019). These errors are not isolated misunderstandings but are deeply rooted in sociocultural contexts. Makonye (2012) describes these errors as arbitrary, occurring when students misinterpret mathematical tasks. These errors reflect individual misconceptions and broader sociocultural influences.

Within sociocultural learning theory, mathematical language errors are viewed as manifestations of deeper conceptual misunderstandings (Тонких, 2023). Mahadewsing et al. (2024) highlight how these errors hinder comprehension, leading to misconceptions and problem-solving errors. Misinterpretation of mathematical language can result in fundamental misunderstandings of principles, impeding meaningful engagement. Preservice calculus teachers face unique challenges related to mathematical language errors, compounded by sociocultural factors. They must possess a deep understanding of mathematical concepts and navigate classroom sociocultural dynamics (Hill et al., 2005; Johnson, 2022). Cultural diversity, social interactions, and learning practices shape students' mathematical experiences and their use of language. Recognizing these dimensions, teachers can develop strategies to address these challenges. Essien (2010) and Khalo et al. (2022) emphasize explicit language instruction, providing clear explanations of terminology and symbols within cultural contexts. Additionally, language-focused activities and peer interaction enhance proficiency.

Mathematical language errors are deeply embedded in sociocultural contexts. Understanding these errors through sociocultural learning theory allows teachers to develop effective strategies, improving language proficiency and overall academic achievement in diverse contexts. Research supports the importance of mathematical language proficiency in teaching and learning. Prince and Frith (2020) found that academic numeracy is closely related to mathematical and language abilities, emphasizing the need for integrating language development into mathematics education.

Error analysis is crucial for understanding and addressing mathematical language errors. Baidoo et al. (2020) state that error analysis helps identify areas where students struggle, allowing targeted interventions. Examining errors provides insights into their causes, whether from linguistic misunderstandings or deeper conceptual issues. Error analysis identifies common misconceptions arising from language difficulties. Makonye and Matuku (2016) found that many errors in solving quadratic equations stem from misunderstandings of terminology and symbols. Addressing linguistic challenges helps students develop a robust understanding, reducing errors and enhancing performance.

Sociocultural factors in language proficiency are well-documented. Johnson (2022) emphasizes that communication practices are influenced by sociocultural contexts. Students' backgrounds, language proficiency, and educational experiences impact their interaction with mathematical language. For preservice teachers, understanding these dynamics is crucial for creating inclusive, supportive environments. Incorporating sociocultural learning theory in education involves recognizing the interconnectedness of language, culture, and cognition. Vygotsky's theory highlights how language and communication are shaped by cultural and social contexts, influencing learning experiences (Johnson, 2022). This perspective is relevant for addressing mathematical language errors, acknowledging that these errors are rooted in broader sociocultural influences. To effectively address these errors, teachers must adopt a holistic approach encompassing linguistic and conceptual dimensions. Essien (2021) advocates for strategies including explicit instruction on terminology and symbols and providing opportunities for students to practice language skills in various contexts. This approach improves understanding and language proficiency. Interactive activities, such as word problems, discussions, and writing tasks, enhance engagement with mathematical language (Nugraha & Prabawanto, 2021). These activities encourage active use of language, improving comprehension and usage. Incorporating these activities creates a dynamic learning environment that supports language development.

Creating a collaborative learning environment that promotes peer interaction is also effective. Peer feedback allows students to identify errors, clarify misconceptions, and refine communication skills through constructive dialogue (Morales López & Tuzón Marco, 2022; Ili, 2022). This approach fosters language proficiency and a sense of community. Mathematical language errors pose significant challenges in teaching and learning. They impede understanding, hinder communication, and lead to misconceptions. Addressing these errors through targeted interventions, training programs, and error analysis is crucial. A holistic approach considering linguistic and conceptual dimensions helps students develop a deeper understanding and improve communication, enhancing academic achievement and fostering an inclusive learning environment.

METHODOLOGY

This study employs an exploratory sequential mixed-method design to investigate mathematical language errors among first and second-year preservice teachers enrolled in mathematics courses at a university. According to Hirose and Creswell (2023), exploratory sequential design implies collecting and analysing qualitative and quantitative data in two consecutive phases within one study. The aim is to understand the nature of mathematical language errors, their impact on academic achievement, and practical strategies to address them. The target population consists of preservice teachers in the early stages of their teacher education journey, focusing on mathematics education.

Sampling and Data Collection

This study employed a stratified purposeful sampling technique, chosen for its ability to ensure representation from both first-year and second-year cohorts of preservice teachers at a specific college. The target population comprised 1001 preservice teachers, stratified based on their academic year. This sampling approach facilitates the identification of common patterns and themes across different stages of preservice teacher education (Fetters & Tajima, 2022). From each stratum (first-year and second-year), 60 students were purposively selected, resulting in a total sample size of 120 participants. All selected preservice teachers completed a calculus assessment, providing quantitative data for the analysis.

Additionally, a subset of 20 preservice teachers from this sample were purposively chosen for in-depth interviews, generating qualitative data for the study. The semi-structured interviews explored participants' experiences with errors in mathematical language, including their perceptions, challenges, and strategies for addressing them. The flexibility of the semi-structured format allowed for depth in participants' responses and the exploration of emerging themes (Hirose & Creswell, 2023).

For the qualitative data, thematic analysis was conducted on the interview transcripts to identify patterns and themes within participants' responses (Oliveira, 2023). This process involved coding the data to uncover recurring themes related to participants' experiences with mathematical language errors. Quantitative data from the calculus assessments were also analyzed using descriptive statistics, chi-square tests, and analysis of variance (ANOVA). These statistical methods were employed to establish possible relationships between mathematical language errors and academic performance among the preservice teachers. The combination of qualitative and quantitative analyses provided a comprehensive understanding of the phenomenon under study, allowing for triangulation of findings and enhancing the validity of the research outcomes. An exploratory sequential mixed method allowed for a rich, multifaceted exploration of mathematical language errors among preservice teachers. By integrating these diverse data sources, the study aimed to provide a nuanced understanding of the challenges and potential interventions related to mathematical language proficiency in preservice teacher education.

Data Presentation and Analysis

Qualitative analysis

Some of the calculus questions analysed for this study are as follows:

$$Question\ 1.2\ Find\ the\ derivative\ of\ the\ function\ f(x) = \frac{d}{dx}(x^2.e^x + \cos(x) + 5)$$

$$Question\ 2.2\ Find\ the\ derivative\ of\ f(x, y) = -3x^2 + y^2$$

$$Question\ 3.4\ Calculate\ the\ value\ of\ f(x) = \int_0^{\pi} sin\ (x)dx$$

The scripts of the students were scrutinised, and the following interview ensued:

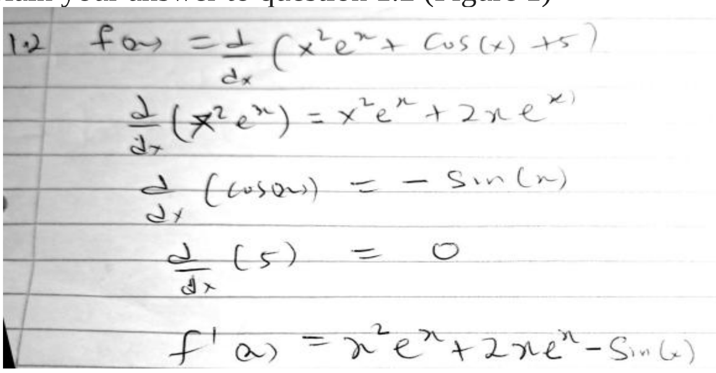
Researcher: Can you explain your answer to question 2.2 to me?

S11: My answer to question 2.2 is $-6x + y^2$.

Researcher: Why not $-3x^2 + 2y$.

S11: Oh, that will be derivative with respect to y. The question was ***not explicit****. Hence, I differentiated with respect to x.*

Researcher: Explain your answer to question 1.2 (Figure 1)



1.2 $f(x) = \frac{d}{dx}(x^2e^x + \cos(x) + 5)$

$\frac{d}{dx}(x^2e^x) = x^2e^x + 2xe^x$

$\frac{d}{dx}(\cos(x)) = -\sin(x)$

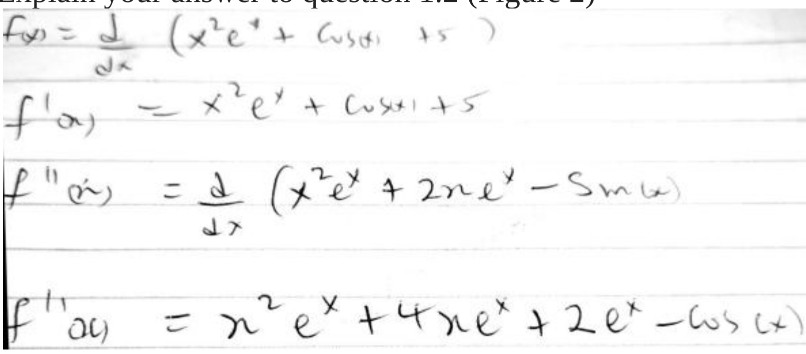
$\frac{d}{dx}(5) = 0$

$f'(x) = x^2e^x + 2xe^x - \sin(x)$

Figure 1: S16 Excerpt

S16: I calculated the derivative of the function f(x).

Researcher: Explain your answer to question 1.2 (Figure 2)



$f(x) = \frac{d}{dx}(x^2e^x + \cos(x) + 5)$

$f'(x) = x^2e^x + \cos(x) + 5$

$f''(x) = \frac{d}{dx}(x^2e^x + 2xe^x - \sin(x))$

$f''(x) = x^2e^x + 4xe^x + 2e^x - \cos(x)$

Figure 2: S20 Excerpt

S20: The expression inside the parentheses $x^2.e^x + cos(x) + 5$ *is already differentiated, so I did the second differentiation of it.*

The ambiguity of the question might have caused the different solutions. The ambiguity can lead to confusion, especially in contexts involving multiple variables.

Additionally, question 2.2, which stated that find the derivative of the function f (x, y) $= -3x^2 + y^2$, was also unclear because it failed to state whether the students should find the partial derivative with respect to x or y, or the total derivative. The ambiguity could have been avoided if the question had complete information. For instance, question 2.2 could have been stated as, find the derivative of f (x, y) with respect to x or y or, finding the derivative of f(x) at x = 2 which clarifies the context and ensures that the intended derivative is understood correctly.

Researcher: Explain your answer to question 3.4 (Figure 3)

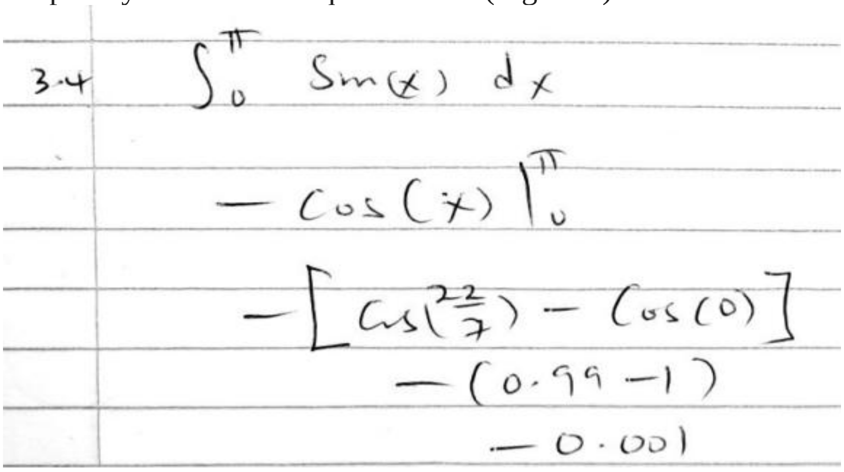


Figure 3: Excerpt S2

S2*: The integral of sin(x) is negative cos(x), then I substituted the limits and calculated the answer*.

Researcher: Explain your answer to question 3.4 (Figure 4)

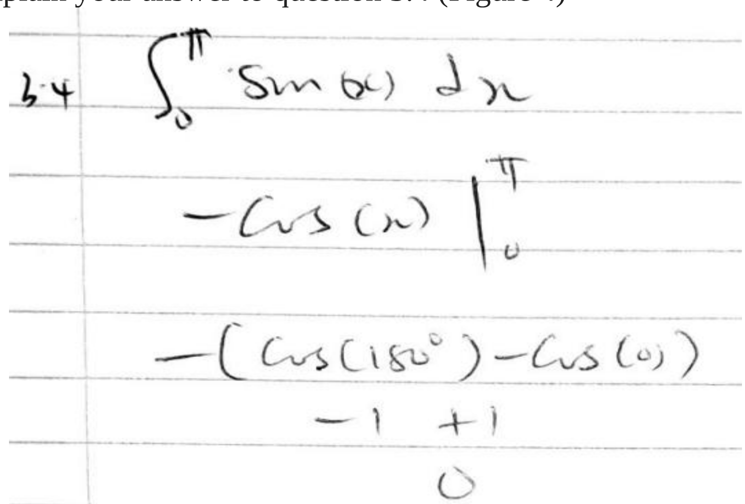


Figure 4: Excerpt S17

S17*: Antiderivative of sin(x) will be negative cos(x), plugged-in the limits. Cos(180^0) will be positive and cos(0^0) will be negative 1, I then multiply through with a negative one, final answer is zero.*
Researcher: Why did you use 180^0 instead π?
S17: *They are the same.*

Participants S2 and S17 both swapped the variable Cos(π) with numerals without substantiating them. This suggests the use of correct terminologies, such as mathematical symbols and corresponding values, when discussing mathematical concepts is essential.

Ambiguous language errors and incorrect terminology errors are emerged themes derived from the transcriptions through a rigorous process of thematic analysis. The transcribed data have been meticulously coded to identify these two themes recurring patterns and significant issues in participants' responses. Frequent references to specific terminology problems and ambiguous language were noted during this coding phase. As the codes were aggregated into broader categories, it became clear that these two themes were the most prevalent and significantly impacted participants' understanding of mathematical concepts. The themes were further refined by continuously comparing and contrasting the data to ensure they accurately reflected the core challenges faced by the participants. This method is consistent with established thematic analysis techniques, which focus on organizing codes into meaningful groups to uncover deeper insights and relationships within the data (Fetters & Chihiro, 2022).

Quantitative analysis

The data presented in Table 1 examines the performance of 120 preservice teachers, split into two cohorts: first-year (60 teachers) and second-year (60 teachers), all undergoing a calculus course.

Table 1: Mathematical Language Errors Dataset

Preservice Teachers	Ambiguous Language error(ALE)	Incorrect Terminology Error(ITE)	MLE	Other Error types	Academic performance
S1(60)	301	159	460	820	1720
S2(60)	150	118	268	466	2266

The dataset encompasses the frequency of Mathematical Language error types made by these preservice teachers during assessments(50 marks), alongside their corresponding academic performance scores. These error categories include incorrect terminology errors and ambiguous language errors. Incorrect terminology errors occur when students incorrectly swap mathematical terms (s) or symbols (s) without substantiating the substitution. Ambiguous language error also occurs when the response from the student is wrong as a result of the ambiguity of the question. Other known error types that were observed and categorised as other error types but not covered in this study are conceptual, procedural, and application errors (Baidoo & Luneta, 2024). Columns 2 and 3 indicate frequencies of incorrect terminology, and ambiguous language for the first and second-year preservice teachers respectively. These two sources of mathematical language error (MLE) are totalled in column 4. The aggregate for other error types (Conceptual, procedural and application errors) was recorded in column 5. The total correct ticks reflect the number of correct marks achieved by preservice teachers on the calculus assessment, serving as an indicator of their overall proficiency in the course were labelled as academic performance and recorded in the last column.

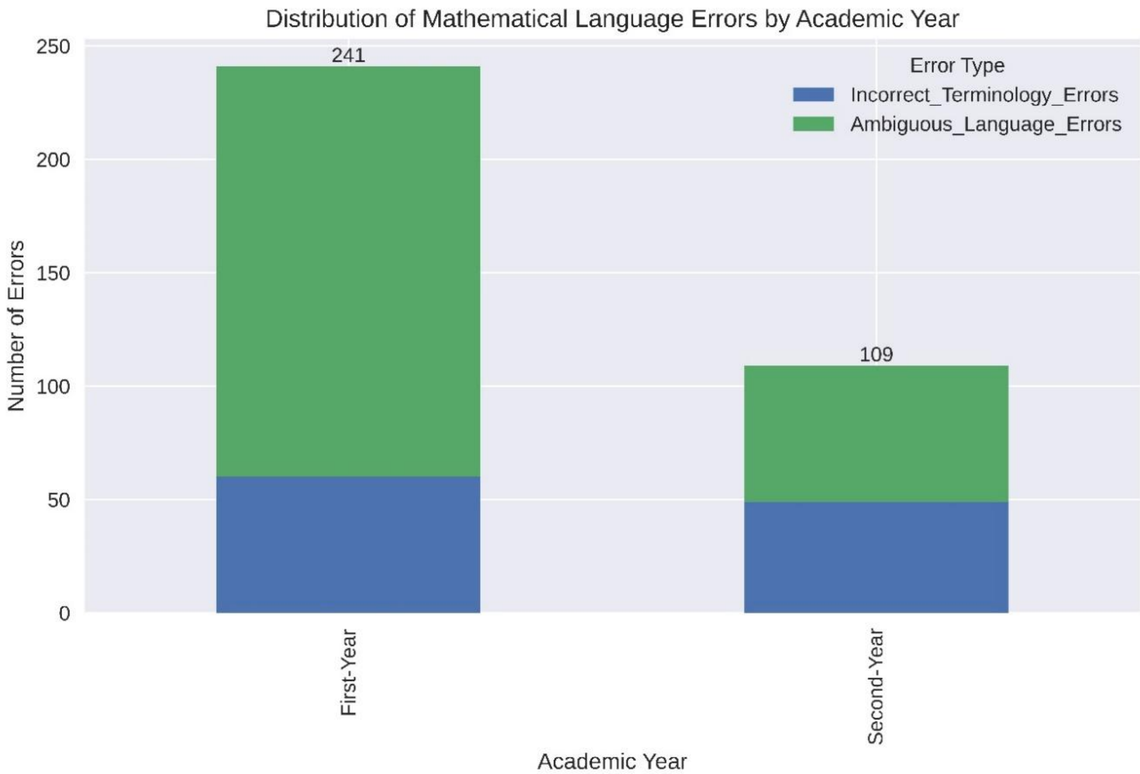


Figure 5: Scatter plot illustrating the relationship between Mathematical Language Errors and Academic year:

Figure 5 scatter plot depicts first-year students have a higher total number of Mathematical Language Errors (241) compared to Second-Year students (109). The majority of errors for first-year students are Ambiguous Language Errors (75.1% of total errors). Second-year students have a more balanced distribution between Incorrect Terminology (44.95%) and Ambiguous Language (55.05%) errors. Despite fewer total errors, second-year students show higher academic performance (40) than first-year students (30). There seems to be an inverse relationship between the total number of errors and academic performance. Both years show a higher number of Ambiguous Language Errors compared to Incorrect Terminology Errors. The reduction in Ambiguous Language Errors from first-year to second-year is particularly notable (181 to 60). These findings suggest that as students progress from their first to second year, they tend to make fewer Mathematical Language Errors overall, with a significant improvement in reducing ambiguous language. This improvement correlates with better academic performance.

Chi-Square Test

A chi-square test was conducted to determine if there is a significant difference in the distribution of error types (Ambiguous Language errors, and Incorrect Terminology errors) between first-year and second-year preservice calculus teachers. The chi-square test is instrumental in identifying whether the types of errors differ significantly between different academic years, thereby indicating potential areas for targeted interventions.

Table 2: Chi-Square Test Results

Statistic	Value
Chi-Square	13.16130
Degree of Freedom	1
P-value	0.0002857

In Table 2, the chi-square test results indicate a statistically significant association between the cohort (First-year versus Second-year) and the type of mathematical language error (Incorrect Terminology versus Ambiguous Language). The extremely low p-value (0.0003) suggests strong evidence against the null hypothesis of no association. This means that the distribution of error types differs significantly between First-year and Second-Year students. First-year students have a higher proportion of Ambiguous Language errors (181 out of 241) compared to Incorrect Terminology errors (60 out of 241). Second-year students have a more balanced distribution between Ambiguous Language errors (60 out of 109) and Incorrect Terminology errors (49 out of 109). This suggests that as students progress from first-year to second-year, they may improve more in avoiding Ambiguous Language errors than in avoiding Incorrect Terminology errors.

ANOVA test

An analysis of variance (ANOVA) test was conducted to examine the impact of different types of mathematical language errors (MLE) on the academic performance of preservice calculus teachers. The ANOVA test results, as shown in Table 3 below, indicate that varying levels of MLE significantly affect academic performance. This significant p-value suggests rejecting the null hypothesis, indicating that MLE levels significantly impact academic performance.

Table 3: ANOVA test result

Source of variation	F-statistic	P-value
MLE, Academic performance	34.475	0.0026

The results highlight a significant inverse relationship between the total number of MLE and academic performance. First-year students exhibited higher frequencies of errors, particularly ambiguous language errors, which correlated with lower academic scores compared to second-year students. These findings emphasize the impact of MLE on academic performance and the need for early and targeted interventions.

By targeting specific error types, such as ambiguous language errors and incorrect terminology errors, educators can prioritize interventions to enhance mathematical language proficiency and the academic success of preservice teachers. The ANOVA results confirm significant differences in the types of mathematical language errors and their impact on academic performance. Tailored educational strategies that address these errors can improve mathematical understanding and academic achievement. Holistic teaching approaches that simultaneously address multiple aspects of mathematical language proficiency can be more effective. Educators should consider integrated strategies that tackle ambiguous language and incorrect terminology to boost academic performance comprehensively.

Discussions

The findings from this study emphasize the significant role of mathematical language proficiency in the academic performance of preservice calculus teachers. Both qualitative and quantitative analyses reveal that mathematical terminology errors and ambiguous language substantially impede understanding and achievement. This aligns with existing literature that underscores the importance of precise mathematical communication for effective learning and teaching (Prince & Frith, 2020).

The qualitative data analysis identified two primary themes: incorrect terminology and ambiguous language. These themes emerged as recurrent issues in participants' responses, indicating widespread challenges in decoding and accurately interpreting mathematical language. Incorrect terminology often leads to fundamental misunderstandings of mathematical concepts. For example, students frequently need to reapply terms, resulting in errors that hindered their problem-solving abilities. Ambiguous language created confusion, making it difficult for students to follow mathematical procedures and arrive at correct solutions.

Quantitative analysis supported these findings, revealing a significant correlation between the frequency of mathematical language errors and academic performance. First-year preservice teachers exhibited higher error frequencies than their second-year counterparts, corresponding to lower academic performance scores. As students advance in their education and receive more targeted instruction, their proficiency in mathematical language improves, leading to better academic outcomes.

The chi-square test results indicated a significant difference in the distribution of error types between first-year and second-year preservice teachers ($\chi2 = 13.16$, $p < 0.05$). This finding highlights the need for early interventions to enhance mathematical language skills.

The ANOVA test further demonstrated that varying levels of mathematical language errors significantly affected academic performance (F-statistic 34.47, $p < 0.05$). These results align with the literature, emphasizing the need for comprehensive strategies to address specific errors to improve overall mathematical understanding (Smith, 2021).

Practical strategies to mitigate mathematical language errors include explicit instruction on mathematical terminology and symbols, incorporating interactive activities that engage students with mathematical language, and promoting peer feedback mechanisms. Explicit instruction helps students understand the precise meaning and application of terms and symbols, reducing the likelihood of errors (Hill et al., 2008). Interactive activities, such as word problems and discussions, encourage active engagement with mathematical language, enhancing comprehension and usage skills (Nugraha & Prabawanto, 2021). Peer feedback allows students to collaboratively identify and correct language errors, fostering a supportive learning environment (Morales et al., 2022).

Addressing the sociocultural dimensions of mathematical language errors is also crucial. Sociocultural learning theory suggests that cultural and social contexts deeply influence language and communication practices (Johnson, 2022). Teachers must consider students' cultural backgrounds, language proficiency, and previous educational experiences when developing strategies to improve mathematical language proficiency. Creating an inclusive classroom environment that values linguistic and cultural diversity can enhance students' engagement and understanding.

Furthermore, this study emphasizes the importance of customized interventions for preservice calculus teachers compared to in-service calculus teachers. Educators can develop more effective teaching strategies by recognizing the interconnectedness of language and mathematical reasoning. For instance, integrating culturally relevant language-focused activities can bridge the gap between students' linguistic backgrounds and mathematical language demands. This approach aligns with the findings of Essien (2010) and Khalo et al. (2022b), who emphasize the importance of contextualizing language instruction within learners' cultural context.

Another critical aspect highlighted by the findings is the role of error analysis in improving mathematical language proficiency. Error analysis provides insights into the specific areas where students struggle, enabling teachers to tailor their instruction to address these challenges. By systematically examining students' errors, teachers can identify patterns and develop targeted interventions that address linguistic and conceptual misunderstandings. This method has proven effective in various studies, including those by Baidoo, Adane, and Luneta (2020) and Makonye and Matuku (2016), who found that error analysis helps pinpoint the root causes of misconceptions and provides targeted support to students.

The study's quantitative analysis also underscores the cumulative effect of mathematical language errors on academic performance. The significant differences in error frequencies and academic outcomes between first-year and second-year students suggest that continued exposure to effective language instruction can substantially improve students' mathematical proficiency. This finding is supported by the results of the t-test and ANOVA, which highlight the need for sustained and focused interventions throughout the preservice teachers' education.

This study highlights the critical impact of mathematical language errors on the academic performance of preservice calculus teachers. Addressing these errors through targeted interventions, language-focused training programs, and comprehensive error analysis is essential for promoting successful mathematics teaching and learning. By adopting a holistic approach that considers both linguistic and conceptual dimensions, educators can help students develop a deeper understanding of mathematical concepts and improve their ability to communicate mathematical thinking accurately. This, in turn, will enhance overall academic achievement and foster an inclusive and supportive learning environment for all students.

The findings underscore the importance of early and sustained interventions to improve mathematical language proficiency. Future research should explore the long-term impact of these interventions on student learning outcomes and identify the most effective strategies for integrating language and mathematical instruction. By continuing to investigate the interplay between language and mathematics, educators can develop more effective approaches to support preservice teachers and their students in achieving academic success.

CONCLUSION

This study highlights the importance of mathematical language proficiency in the academic performance of preservice calculus teachers. Errors in mathematical terminology and ambiguous language significantly hinder understanding and achievement. Both qualitative and quantitative analyses reveal a strong correlation between these errors and academic outcomes, suggesting that improving language skills is essential for educational success. First-year preservice teachers show higher error frequencies than second-year students, resulting in lower performance scores.

Effective strategies to address these issues include explicit instruction on terminology and symbols, interactive activities, and peer feedback, all of which should be implemented with careful consideration of sociocultural dimensions. Explicit instruction should acknowledge diverse linguistic backgrounds, incorporating culturally responsive teaching methods that build upon students' existing knowledge. Interactive activities should be designed to promote collaborative learning and cultural exchange, allowing students to share unique perspectives on mathematical concepts within various cultural contexts. Peer feedback sessions should foster a supportive and inclusive environment that values diverse communication styles while promoting the use of standard mathematical language.

Addressing sociocultural dimensions of language is crucial for creating an inclusive learning environment. By incorporating these culturally sensitive strategies, teacher training programs can better prepare preservice teachers to overcome language-related challenges in mathematics education and create more equitable classrooms. Additionally, future studies could investigate the role of technology in supporting mathematical language acquisition and the potential for adaptive learning platforms to provide personalized feedback on language use. These technological approaches should be designed with cultural sensitivity in mind, ensuring they cater to diverse learning styles and backgrounds.

A potential drawback of implementing these sociocultural strategies is the challenge of balancing standardized mathematical language with the preservation of culturally specific mathematical knowledge. Further research could explore how to effectively integrate diverse mathematical practices and terminologies while maintaining clarity and consistency in mathematical communication across cultures. By addressing these areas, teacher training programs can enhance their effectiveness in preparing preservice teachers to navigate the complex interplay between mathematical language, cultural diversity, and effective pedagogy.

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Limitations of this tudy

This study, while providing valuable insights into the impact of mathematical language proficiency on the academic performance of preservice calculus teachers, is subject to several limitations. First, the sample size was relatively small and restricted to a specific geographic region, which may limit the generalizability of the findings to a broader population. Additionally, the study primarily relied on self-reported data and qualitative assessments, which could introduce bias and affect the accuracy of the results. The study's cross-sectional design also limits the ability to establish causal relationships between mathematical language errors and academic performance. Future research should consider longitudinal designs and more prominent, diverse samples to validate and extend the findings. Finally, while the study addressed several key aspects of mathematical language proficiency, other factors, such as students' prior mathematical knowledge and socio-economic background, needed to be thoroughly explored, which could also influence academic outcomes.

Disclosure Statement

The author reported no potential conflict of interest.

Institutional Review Board Statement: The Ethical Committee of the University of Johannesburg, South Africa,, approved this study on 19 August 2021 (Ref. No. Sem 2-2021-044).

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